Directional Gain
of IEEE 802.11 MIMO Devices
Employing Cyclic Delay Diversity

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Technical Research Branch
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ACKNOWLEDGMENTS

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EXECUTIVE SUMMARY

This report documents the theoretical modeling behind the formulas employed by the Federal Communications Commission (FCC) Laboratory for determining directional gain of radio transmissions from IEEE 802.11-compliant wireless local area network (LAN) devices when transmitting on multiple antennas in modes employing cyclic delay diversity (CDD). The new formulas are included in the guidance for computing directional gain in Knowledge Database (KDB) Publication 662911, "Emissions Testing of Transmitters with Multiple Outputs in the Same Band (e.g., MIMO, Smart Antenna, etc)".[1]

Many of the FCC's technical rules for "intentional radiators", *i.e.*, devices that intentionally transmit signals on radio frequencies, specify limits on maximum total transmit power and on maximum power spectral density that are dependent on the directional gain associated with those transmissions. This is true of the rules under which most IEEE 802.11-compliant wireless LAN devices operate (sections 15.247 and 15.407 of the FCC Rules)[2] as well as a number of other rules.

For devices that have a single transmitter output driving an antenna, the directional gain is simply that of the antenna. However, devices employing multiple input multiple output (MIMO) technology can achieve additional directional gain, called *array gain*, beyond the gains of the individual antennas by transmitting signals that are mutually correlated. In some cases this additional gain is intentionally created to improve performance, as is the case when transmit beamforming is employed. In other cases, as with cyclic delay diversity (CDD), the array gain is not intentional.

Previous FCC guidance for determining directional gain of MIMO transmissions required that the maximum array gain that is theoretically possible for correlated transmissions be assumed whenever the transmissions exhibited any mutual correlation. That maximum possible array gain is given by *10 log(N_{ANT})* dB, where N_{ANT} is the number of transmit antennas. In some cases this approach was known to overestimate array gain, resulting in greater reductions in transmit power than would be required if a more accurate way of computing array gain were available.

This report describes the development by the FCC Laboratory of a more accurate way to estimate array gain for the specific case of cyclic delay diversity (CDD) transmissions by devices operating under the IEEE 802.11 standard. The new techniques are based on theoretical calculations of beam patterns and the resulting array gains for twelve sample antenna array configurations with simulated transmissions using the CDD delays specified in the IEEE 802.11 standard. Based on the modeled array gain results as a function of transmit signal bandwidth, the new approach requires the continued use of the *10 log(N_{ANT})* formula when computing *narrowband* array gain for determining the required reductions in power spectral density of transmissions. However, a new formula computes *broadband* array gain as either 0 dB or 3 dB depending on transmit signal bandwidth and number of transmit antennas. The broadband array gain value is used to compute array gain for determining required reductions in transmit power.

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[1] Available at www.fcc.gov/labhelp by clicking "Major Guidance Publications" or by clicking "Advanced KDB Search" and entering the publication number. The array gain computations described in this technical report were implemented in version v01r01 of KDB Publication 662911, dated September 26, 2012.

[2] The FCC rules are found in Title 47 of the Code of Federal Regulations (abbreviated as 47 CFR), available at http://www.ecfr.gov. Sections 15.247 and 15.407 are contained within Part 15, which governs unlicensed radio frequency devices.

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INTRODUCTION

This report documents the theoretical modeling behind the formulas employed by the Federal Communications Commission (FCC) Laboratory for determining directional gain of radio transmissions from IEEE 802.11-compliant wireless local area network (LAN) devices when transmitting on multiple antennas in modes employing cyclic delay diversity (CDD). The new formulas are included in the guidance for computing directional gain in Knowledge Database (KDB) Publication 662911, "Emissions Testing of Transmitters with Multiple Outputs in the Same Band (e.g., MIMO, Smart Antenna, etc)".[1]

BACKGROUND

Directional Gain in the FCC Rules

Many of the FCC's technical rules for "intentional radiators", *i.e.*, devices that intentionally transmit signals on radio frequencies, specify limits on maximum total transmit power that are dependent on the directional gain associated with those transmissions. For example, the rules under which most IEEE 802.11-compliant wireless LAN devices operate are found in sections 15.247 and 15.407 of the FCC Rules.[2] (Section 15.407 is contained in Part 15, Subpart E, "Unlicensed National Information Infrastructure Devices", which is commonly abbreviated as U-NII.) Both 15.247 and 15.407 specify the maximum transmit power ("maximum conducted output power") permitted for devices operating under those rule sections. For sufficiently broadband signals, the maximum power levels range from 50 milliwatts to 1 watt depending on band of operation for systems using digital modulation and satisfying the other requirements of the rules.[3] Lower limits apply in the U-NII band when the emission bandwidth is less than 20 MHz.[4] Both rule sections also specify that the power must be reduced below those stated values "if transmitting antennas of directional gain greater than 6 dBi are used."[5] (Different requirements apply for systems that are used exclusively for fixed, point-to-point operation and for certain other categories of systems.[6]) The amount of power reduction required depends on the directional gain and is specified in the rules.

The rules also specify limits on the maximum power spectral density of the transmitted signals. For example, section 15.407 of the rules specifies limits on the power spectral density in terms of the maximum power permitted within any 1 MHz band. Like the limits on total transmit power, the power spectral density limits are reduced if the directional gain of the transmitting antennas exceeds 6 dBi.[7]

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[1] Available at www.fcc.gov/labhelp by clicking "Major Guidance Publications" or by clicking "Advanced KDB Search" and entering the publication number.

[2] The FCC rules are found in Title 47 of the Code of Federal Regulations (abbreviated as 47 CFR), available at http://www.ecfr.gov. Sections 15.247 and 15.407 are contained within Part 15, which governs unlicensed radio frequency devices.

[3] See 47 CFR 15.247(b)(3) and 15.407(a).

[4] Power limits in 15.407(a) are specified in the form *X or Y dBm + 10 log(B)*, whichever is less, where B is the emission bandwidth in MHz and X ranges from 50 mW to 1 watt. The parameters are selected such that if B < 20 MHz, *Y dBm + 10 log(B)* yields the lower result.

[5] See 47 CFR 15.247(b)(4) and 15.407(a).

[6] See 47 CFR 15.247(b)(4), 15.247(c), and 15.407(a).

[7] See 47 CFR 15.407(a).

IEEE 802.11 operations for certain licensed applications may be permitted under other FCC rule sections, including:[8]

- Part 90M—Intelligent Transportation Systems Radio Service, sections 90.371-90.383, Dedicated Short-Range Communications Service (DSRCS);
- Part 90Y—Regulations Governing Licensing and Use of Frequencies in the 4940-4990 MHz Band, sections 90.1201-90.1217 (limited to public safety use);
- Part 90Z—Wireless Broadband Services in the 3650-3700 MHz Band, sections 90.1301-90.1337; and,
- Part 95L, Dedicated Short Range Communications Service On-Board Units (DSRCS-OBUs), sections 95.1501-95.1511, with power limits specified in section 95.639;

In Part 90Y, the rules specify limits on conducted output power and power spectral density which must be reduced based on directional antenna gain, similarly to 15.407. In some other cases (*e.g.*, Part 90M and 90Z), reductions based on directional antenna gain are not explicitly specified because the rules specify power in terms of equivalent isotropically radiated power (EIRP). Since EIRP is typically calculated from transmit power and directional antenna gain, the use of directional antenna gain in determining the power limit is implicit in the rule.

Meaning of Directional Gain

Directional gain of a transmitting antenna system is a measure of the power that would be received from the antenna at a point in the far field in the direction of highest emission, relative to the power that would have been received had the transmitting antenna had an isotropic emission pattern (*i.e.*, equal transmission intensity in all directions). Thus, directional gain is a power ratio. It is usually expressed in dBi (decibels relative to an isotropic radiator) by computing $10 \log_{10}$ of the power ratio.

Array Gain

A transmitter that employs multiple transmit antennas can achieve directional gain greater than the gain of its individual antennas. At a given receive location, the signals from the various transmit antennas may combine constructively or destructively. If they combine constructively, additional directional gain, known as array gain, is achieved.

In particular, if the same signal is fed to each transmit antenna but with appropriate relative delays, the signals received in a far field location in a particular direction may be aligned in time resulting in a received voltage that is N_{ANT} times the voltage that would have been received from an individual antenna, where N_{ANT} is the number of transmit antennas. In that case the received power would be N_{ANT}^2 times the power that would have been received from an individual antenna because power is proportional to the square of voltage. On the other hand, if the transmitted signals are independent of one another, they do not combine constructively and the total received power would be simply N_{ANT} times the power that would have been received from an individual antenna. Thus, by transmitting signals that are identical but delaying them appropriately so that they align in time in a particular direction, an array gain of $N_{ANT}^2/N_{ANT} = N_{ANT}$ is achieved. In decibels, the additional gain is $10 \log_{10}(N_{ANT})$ dB.

Array gain can be achieved intentionally as in the process of forming transmit beams. It may also occur unintentionally as a result of transmitting mutually correlated signals on multiple antennas. In either case, the array gain adds to the directional gain of the individual antennas and must be included as part of the total directional gain that is used to compute required reductions in the transmit power limits.

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[8] Table D-1 "Regulatory requirement list", Appendix D of IEEE 802.11-2012, "Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications"

CYCLIC DELAY DIVERSITY (CDD) IN IEEE 802.11

Cyclic delay diversity (CDD), also known as cyclic shift diversity (CSD), is a method for achieving transmit diversity in order to improve performance of radio communication systems that employ multiple transmitter outputs. CDD transmits the same signals from two or more antennas, but with a different cyclic delay applied to each transmit stream.

The technique was introduced to IEEE 802.11 communications in the 802.11n version of the standard, which defined the delay values to be employed for up to four transmit antennas. The upcoming 802.11ac standard extends the list of specified delays for up to eight transmit antennas. In both cases the standards defined two different sets of delays to be used: Table 1 shows the delays for non-High-Throughput (non-HT) fields and Table 2 shows the delays for High-Throughput (HT) fields.

Table 1. Short Cyclic Delays Adopted by IEEE 802.11 for Non-High-Throughput Fields

N_{ANT}	Delay (ns) for Specified Transmit Chain Number							
	1	2	3	4	5	6	7	8
2	0	200						
3	0	100	200					
4	0	50	100	150				
5	0	175	25	50	75			
6	0	200	25	150	175	125		
7	0	200	150	25	175	75	50	
8	0	175	150	125	25	100	50	200

Values are from Table 22-10 of IEEE Draft P802.11ac_D3.0. The final 802.11ac standard had not been approved as of this writing. The values in rows corresponding to two through four antennas (i.e., transmit chains) correspond to Table 20-9 of IEEE Standard 802.11-2012, which was adopted as part of the 802.11n revision.

Table 2. Long Cyclic Delays Adopted by IEEE 802.11 for High-Throughput Fields

N_{ANT}	Delay (ns) for Specified Transmit Chain Number							
	1	2	3	4	5	6	7	8
2	0	400						
3	0	400	200					
4	0	400	200	600				
5	0	400	200	600	350			
6	0	400	200	600	350	650		
7	0	400	200	600	350	650	100	
8	0	400	200	600	350	650	100	750

Values are from Table 22-11 of IEEE Draft P802.11ac_D3.0. The final 802.11ac standard had not been approved as of this writing. The values in rows corresponding to two through four antennas (i.e., transmit chains) correspond to Table 20-10 of IEEE Standard 802.11-2012, which was adopted as part of the 802.11n revision.

Though one reason for using CDD can be to prevent unintentional beamforming,[9] CDD can create an unintentional beamforming effect of its own that focuses transmit energy in specific directions. CDD is,

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[9] Ho Huat Peh, Sumei Sun, Patrick Ho-Wang Fung and Chin Keong Ho, "Frequency-Domain Timing Synchronization for IEEE 802.11n Communications Systems", 2011 IEEE 22nd International Symposium on Personal Indoor and Mobile Radio Communications, p.859.

in fact, very similar to the techniques used for transmit beamforming except that, with CDD, the delays are cyclic and are longer than the delays used for beamforming.

The delays are applied cyclically within Fast Fourier Transform (FFT) window used in creating the Orthogonal Frequency Division Multiplexing (OFDM) modulated signal meaning that the delays wrap around from the end of the window to the beginning. For IEEE 802.11 OFDM, the window duration is 3.2 μs,.[10] Because the cyclic delays of 25 to 750 ns are relatively small (from 0.8 to 23.4 percent of the window duration), the wrap around is ignored in the analysis presented in this report.

The cyclic delays are generally much larger than the delays that would be used for intentional beamforming. For example, consider an array of five transmit antennas spaced at equal intervals of Δx along a line, where $\Delta x = 6$ cm (approximately half wavelength based on 2.44 GHz operating frequency). If the same transmit signal is delivered to all four antennas without delays, a transmit beam will be formed with peak response in a direction broadside to (i.e., perpendicular to) the line along which the antennas are placed. If, however, the signal is delayed by amounts of 0, d, 2d, 3d, and 4d, respectively, at the five antennas and if d is less than or equal to the propagation time between adjacent antennas ($\Delta x/c = 0.2$ ns, where c is the speed of light), the beam will be steered off broadside. Thus, for intentional beamforming in this example the maximum relative delay between signals applied to adjacent antennas would be 0.2 ns, and the maximum delay between the signals applied to the outermost pair of antennas would be 0.8 ns. By contrast, the cyclic delays range from 25 to 650 ns.

Because of the large size of the delays, cyclic delay diversity creates an unintentional beamforming effect that is frequency dependent, i.e., the pointing directions of the beams vary with frequency. The array gain may be high when viewed over a narrow frequency range (such as for compliance with FCC power spectral density limits), but the gain is effectively reduced when viewed over a broad frequency range (such as for compliance with FCC power limits) due to smearing of the beam pointing direction.

MODELING

Array gain, whether created intentionally by beamforming or unintentionally by cyclic delay diversity, adds to the total directional gain of the antenna system, and therefore affects the maximum total transmit power and power spectral density that FCC permits under its rules. High array gain can result in a need to reduce transmit power in order to comply with FCC rules.

The analysis presented in this report was performed to permit more accurate computation of broadband array gain associated with cyclic delay diversity implementation in IEEE 802.11 compliant transmitters. The analysis forms the basis for FCC Laboratory guidance on array gain calculations for testers of such transmitters.[11] (Previous guidance had required that array gain be computed as $10 \log_{10}(N_{ANT})$, which overestimates array gain in many cases.)

For this report, broadband array gain calculations were performed on twelve array configurations as a function of frequency and bandwidth over one of two 100 MHz spans of center frequencies.

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[10] $T_{DFT} = 3.2$ μs, as specified in Table 20-6 of IEEE 802.11-2012, "Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications." The cyclic delays are applied before the addition of a guard interval of 400 or 800 ns.

[11] The guidance developed from this analysis was introduced in the September 26, 2012 revision of the FCC Laboratory's Knowledge Database (KDB) publication number 662911, "Emissions Testing of Transmitters with Multiple Outputs in the Same Band (e.g., MIMO, Smart Antenna, etc)". The document is available at www.fcc.gov/labhelp by clicking "Advanced KDB Search" and entering the publication number, 662911, into the publication number search field. A link to the document is also provided at the FCC's Office of Engineering and Technology (OET) major guidance publications page: https://apps.fcc.gov/oetcf/kdb/reports/GuidedPublicationList.cfm.

Array Configurations for FCC Analysis of 802.11 Cyclic Delay Diversity

Figure 1 shows the geometric configurations of antenna arrays that were modeled for this study. In some cases, the configurations were chosen to match existing 802.11 products. In others, hypothetical configurations were created.

Each array is contained in a horizontal plane. Each of the twelve configurations include two, four, five, or eight antennas arranged in a line, a rectangle, a square, or a circle. The reason for including configurations with both four and five antennas is that the cyclic delays specified for non-High-Throughput fields (Table 1) change from being multiples of 50 ns for four or fewer antennas to being multiples of 25 ns for five or more antennas, and this change has a significant effect on the broadband array gains that can occur for 20 MHz channels. For four, five, or eight antennas, two rectangular or square array configurations are included that are identical except for the numeric ordering of the antenna elements; the numeric ordering, which determines which delays are applied to which antennas, has a significant effect on the modeled beam patterns.

Methodology

The analysis was performed using two MATLAB programs developed by the author specifically for this project and shown in Appendix A. Both programs compute beam patterns by summing phasors from each antenna with phase shifts corresponding to the sum of two time delays: the cyclic delay for the signal transmitted by the antenna and the relative propagation delay from that antenna (based on antenna array geometry) in the direction being evaluated. In performing the calculations, the individual antennas are assumed to have isotropic patterns.

The first program simply generates visualizations of narrowband or broadband beam patterns as a function of azimuth at zero degrees elevation angle. The second program performs the following functions:
- Computes frequency-dependent beam patterns over a span of 0-360° azimuth and 0-45° elevation;[12]
- Smears patterns over bandwidths ranging from 0 – 80 MHz;
- Performs those computations for center frequencies over a specified frequency range;
- Finds the peak response (array gain) for each bandwidth and center frequency,
- Computes the maximum and median array gains across a 100 MHz wide span of center frequencies.

All analysis assumes that no spatial multiplexing is performed, *i.e.*, that the transmissions consist of only a single spatial stream.[13]

RESULTS

Though the IEEE 802.11 standard defines specific channels of operation, the analysis for this report was performed over a continuous frequency range of center frequencies in order to provide a better understanding of the frequency dependence of the results and to reduce the impact of using fixed antenna spacings in each modeled array configuration.[14] All analysis was performed over a 100 MHz span of

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[12] Because the arrays were planar, the beam patterns are symmetrical about 0° elevation angle; consequently, there was no need to compute results for negative elevation angles. The analysis was limited to an upper elevation angle of 45° because the gain of a typical array implemented with vertical dipole antennas would fall off with increasing elevation angle due to the patterns of the individual antennas.

[13] With "pure" spatial multiplexing, each antenna transmits an entirely different, independent data stream as a means of increasing data throughput. If all antennas carry different data streams, then the transmissions among the various antennas are mutually uncorrelated and no array gain occurs. If the number of spatial streams that is transmitted is less than the number of transmit antennas, spatial multiplexing or cyclic delay diversity or a combination of the two can be used to create the transmit signals for the "extra" antennas. This results in lower array gain than the cases modeled in this report.

[14] When measured in wavelengths, the spacing between antennas varies with frequency.

center frequencies from 5700 to 5800 MHz.[15] In addition, limited analysis was performed over a 100 MHz span from 2400-2500 MHz to confirm that the higher frequency results are applicable to other bands.

Narrowband Beam Patterns

Figure 2(a) shows narrowband azimuth beam patterns at zero-degrees elevation angle as a function of frequency for the four-antenna array identified in Figure 1 as Rectangle-2 for transmissions with short cyclic delays (Table 1). Figure 2(b) shows the equivalent pattern for transmissions with long cyclic delays (Table 2). Azimuth is plotted on the vertical axis, and frequency (100 MHz span from 2400 to 2500 MHz) is plotted on the horizontal axis.[16] Gain is shown as color on a scale that ranges from -3 dB to +6 dB, the theoretical maximum array gain for a four-antenna array.

By looking along vertical cross-sections through the graphs, one can see that multiple peaks in gain (i.e., multiple beam lobes) occur over the 360 degree span of the graph. Multiple lobes are, in fact, common when the spacing between adjacent antennas exceeds a half wavelength. (In the center of the plotted frequency range, the antenna spacing is 0.9 wavelengths in one dimension and 1.4 wavelengths in the other.)

Additionally, the locations of the peaks vary with frequency, and those changes with frequency are more rapid when long cyclic delays are applied as opposed to short cyclic delays, as can be seen by comparing Figure 2(b) to Figure 2(a). When measured across the full bandwidth of a transmission, the variation with frequency can effectively smear the beam patterns, causing lower effective broadband array gain.

Figure 3 shows narrowband azimuth beam patterns equivalent to those of Figure 2 except that they are plotted across a frequency span from 5700 to 5800 MHz. Because of the higher frequencies, the antenna spacings measured in wavelengths (2.1 wavelengths in one dimension and 3.4 wavelengths in the other in the center of the plotted frequency range) are greater than for the lower frequency range, which results in more beam lobes across 360 degrees of azimuth in Figure 3 than in Figure 2. However, we will show later that the amplitudes of the peaks in the beam pattern are essentially identical among the plots in the two frequency spans and for the two sets of cyclic delays.

Figure 4, which occupies four pages, shows narrowband azimuth beam patterns versus frequency across the 100 MHz span from 5700 to 5800 MHz for the remaining eleven array configurations for both short and long cyclic delays. The structures of the beam patterns are different with each array configuration, but the peaks of all of the patterns can be seen to vary with frequency. In addition the amplitudes of the highest peaks in gain vary with the number of antennas in the array. Consequently, different color scale ranges were used when plotting patterns of configurations having different numbers of antennas: -3 to +3, +6, +7, and +9 dB for arrays with two, four, five, and eight antennas, respectively.

Broadband Beam Patterns

If one averages the beam patterns across the bandwidth of a transmission, the amplitudes of the peaks associated with pattern are reduced. For example, Figure 5 shows the azimuth patterns corresponding to those in Figure 3 after averaging the patterns across the 16.6-MHz bandwidth of an 802.11a transmission. The results are plotted as a function of center frequency of the transmission. The broadband gain remains well below 1 dB at all azimuth angles and center frequencies, whereas narrowband gain exhibited multiple peaks near 6 dB in amplitude.

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[15] This span was selected arbitrarily from the 5 GHz bands that fall under the U-NII rules.

[16] IEEE 802.11 operates in the 2400-MHz band under section 15.247 of the FCC rules, which limits operating frequencies to an 83.5 MHz wide span from 2400 to 2483.5 MHz. However, a 100-MHz wide frequency span was plotted to enable comparison to subsequent plots over a 100 MHz wide span in the 5-GHz bands.

Array Gain Versus Frequency

Presented above were 24 narrowband, frequency-dependent, azimuth beam patterns corresponding to 12 array configurations and two sets of cyclic delays in addition to two broadband, frequency-dependent, azimuth beam patterns for a single array configuration and a single bandwidth. Now we expand each of the 24 narrowband cases to include elevation angle in addition to azimuth and we compute *broadband* patterns as a function of bandwidth and center frequency. In this section we do this for just two bandwidths: zero and 16.6 MHz. In the next section we will expand to a wide range of bandwidths.

For each case (12 array configurations and two sets of cyclic delays), the frequency-dependent beam pattern in azimuth and elevation angle is computed. The patterns are then averaged across the specified bandwidth (zero and 16.6 MHz) as power ratios—before converting to decibels.

The steps described above added two more dimensions (elevation angle and bandwidth) to an already complicated problem. However, after the averaging step, we no longer need to know the pattern as a whole but only the peak value of it because directional gain and array gain are defined as gain in the direction having the highest gain. Hence, after averaging across frequency, we can eliminate the azimuth and elevation dimensions of the problem. We are left with array gain values that are a function of: number of transmit antennas, array configuration, cyclic delay set, center frequency, and bandwidth.

Figure 6 shows array gain versus frequency over a span from 5700 to 5800 MHz for the two-antenna configuration from Figure 1. The upper graph (a) and lower graph (b) are for transmissions with short cyclic delays (Table 1) and long cyclic delays (Table 2), respectively. Each graph includes curves for two bandwidths: zero (narrowband) and 16.6 MHz (the bandwidth of an IEEE 802.11a or g transmission). For this two-antenna configuration, array gain as a function of frequency is constant. Narrowband gain is 3 dB, which corresponds to $10 \log(2)$, the maximum array gain that can occur with two antennas. Broadband array gain across a 16.6-MHz bandwidth is much lower, 0.35 dB for short cyclic delays and 0.17 dB for long cyclic delays.

Figure 7 contains a similar pair of graphs for the four-antenna configurations from Figure 1. Each graph includes curves for three array configurations (line, rectangle-1, and rectangle-2) for both zero and 16.6-MHz bandwidths. Array gain as a function of frequency is essentially constant for only two cases: the line array when transmitting with short cyclic delays and rectangle-2 when transmitting with long cyclic delays, both of which have narrowband array gains of 6.0 dB. In each of the other cases four cases, narrowband array varies as a function of frequency from a low of 3.0 to 3.7 dB to a high of 6.0 dB. The 6.0 dB maximum matches $10 \log(4)$, the maximum array gain that can be achieved by four transmit antennas. When short cyclic delays are transmitted, the maxima occur at intervals of 10 MHz; consequently, two peaks occur every 20 MHz, the reciprocal of 50 ns. Note that 50 ns is the largest common submultiple of the cyclic delays for the four-antenna case for short cyclic delays (Table 1). When long cyclic delays are transmitted, the maxima occur at much closer intervals—1.667 MHz for the line array and 1.25 MHz for rectangle-2. Thus, an integer number of peaks always occurs every 5 MHz. Note that 5 MHz is the reciprocal of 200 ns, which is the highest common submultiple of the long cyclic delays for four antennas (0, 400, 200, and 600 ns). Broadband array gain across a 16.6-MHz bandwidth among the six cases ranges from 0.35 to 0.78 dB, with peaks occurring at intervals matching those of the narrowband array gain.

Figure 8 and Figure 9 show the array gain versus frequency over a span from 5700 to 5800 MHz for the five-antenna and eight-antenna configurations, respectively, from Figure 1. These curves are considerably more complex than the results for smaller numbers of antennas. In all cases, the array gain varies with frequency. In all but one case the narrowband array gain exhibits maxima that match the theoretical maximum possible gains, $10 \log(N_{ANT})$ dB (*i.e.*, 7 dB for $N_{ANT} = 5$ and 9 dB for $N_{ANT} = 8$) within 0.1 dB. However, for the eight-antenna circular array the narrowband array gain was below the

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theoretical 9 dB theoretical maximum by 0.5 dB for short cyclic delays and 0.7 dB for long cyclic delays. For short cyclic delays the maximum narrowband array gain is achieved at intervals of 40 MHz, though in some cases two such peaks with unequal spacings occur within each 40 MHz interval. We note that for short cyclic delays with five to eight antennas, the largest common submultiple of the cyclic delays is 25 ns, the reciprocal of 40 MHz. For long cyclic delays the maximum array gain (narrowband or wideband) is achieved at intervals of 20 MHz. We note that for long cyclic delays with five to eight antennas, the largest common submultiple of the cyclic delays is 50 ns, the reciprocal of 20 MHz. Maximum broadband array gain is achieved at the same intervals as the maximum narrowband array gain but is much lower and exhibits less variation.

For the twelve array configurations and both cyclic delay sets, the following observations apply:
- Array gain is constant as a function of frequency in only four of the 24 cases evaluated.
- The maximum *narrowband* array gain as a function of frequency was close to 10 log(N_{ANT}) dB in each of 24 cases (within 0.7 dB for the eight-antenna circular array cases with both cyclic delays and within 0.1 dB for the other 22 cases).
- *Broadband* array gain at 16.6-MHz bandwidth is considerably lower than the narrowband array gain.
- Both the narrowband and broadband (16.6-MHz bandwidth) array gains appear to be exactly or almost exactly periodic in frequency with periods as follows[17]:
  ◊ 20 MHz or a submultiple thereof for $N_{ANT} \leq 4$ with short cyclic delays (0 or 16.6 MHz bandwidth)
  ◊ 40 MHz for $N_{ANT} \geq 5$ with short cyclic delays (0 or 16.6 MHz bandwidth)
  ◊ 20 MHz for any N_{ANT} with long cyclic delays.
- The apparent periodicities suggest that the 100 MHz span of center frequencies that was analyzed (5700-5800 MHz) is sufficient to characterize the behavior over a much wider span.

As a test of the last bullet, a center frequency span from 2400 to 2500 MHz was analyzed for one selected array (square-1 with five antennas) with both short and long cyclic delays.[18] Figure 10 compares the results for the two frequency spans by shifting the results for 5720 to 5800 MHz downward by a multiple of 40 MHz (the apparent period) to overlay the lower 80 MHz of the 2400-2500 MHz plot. This permits comparison of the results in the two frequency spans based on the apparent 40 MHz periodicity that had been observed in the 5700-5800 MHz span.[19]

With short cyclic delays the narrowband array gain observed in each frequency span appears to be a complicated but periodic function of frequency with a period of 40 MHz. However differences between the two bands, which are separated by 83 x 40 MHz, show that the function is not truly periodic. The array gain in the span from 5720 to 5800 MHz exhibits peaks at 5 MHz intervals—eight such peaks in each 40-MHz "period". The 2400 to 2500 MHz band also exhibits peaks at 5 MHz intervals except that every eighth peak is missing—replaced by a valley (0.4 dB gain). Of the remaining seven peaks in each 40-MHz "period", four are essentially identical between the two bands, two others match within 0.1 dB between bands, and the seventh (and smallest) exhibits a band-to-band difference of slightly less than 0.2 dB. Both bands exhibit two matching peaks of 6.9 dB in each 40 MHz interval. In addition, the higher band exhibits one peak in each 40 MHz having the maximum possible array gain of 7.0 dB.[20]

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[17] RMS difference between narrowband array gain values in dB for points separated by the "period" (20 or 40 MHz) ≤ 0.035 dB. Worst case lack of periodicity occurs for $N_{ANT} = 5$, square-1. Most array-delay combinations have RMS difference between narrowband array gain values in dB for points separated by the "period" (20 or 40 MHz) ≤ 0.002 dB. The corresponding differences for broadband array gain are smaller.

[18] The five-antenna square-1 array was chosen because it exhibits significant, complicated variations in array gain with frequency for both the narrowband and broadband (16.6 MHz) cases and because it exhibited the greatest RMS deviation from periodicity over the 5700-5800 MHz span, as discussed in footnote 17.

[19] The 5700-5800 MHz array gain appeared to have a 40 MHz period when short cyclic delays are transmitted and 20 MHz period when long cyclic delays are transmitted.

[20] Maximum possible array gain = 10 log(N_{ANT}) dB.

With short cyclic delays, the broadband gain at 16.6 MHz bandwidth also appears to be periodic with 40 MHz period within each plotted frequency span, but a discrepancy between bands exists across 7.4 MHz of each 40 MHz span, demonstrating that the periodicity is not maintained across the 3320 MHz frequency difference between the two plotted spans. The curves are, however, indistinguishable (within 0.02 dB) across 32.6 MHz of each 40 MHz span, and the maximum gains in the two spans are the same, 2.97 dB.

The band-to-band comparisons with long cyclic delays are similar to those with short cyclic delays. With long delays, the narrowband array gain in the 5700-5800 MHz frequency span appears to be complicated but periodic function of frequency with a period of 20 MHz and with peaks of varying amplitudes at intervals of 1.25 MHz except that one of the 1.25-MHz intervals in every 20 MHz has a valley rather than a peak. In the 2400-2500 MHz span the array gain versus frequency is similar to that in the higher band except that valleys occur in three of the 1.25-MHz intervals in each 20 MHz period. However, the 12 peaks in each 20 MHz period that are common between the two bands are closely aligned in frequency and amplitude, and maximum narrowband gain is within 0.2 dB of the theoretical maximum gain of 7.0 dB and differs by only 0.1 dB between the two bands. The broadband (16.6 MHz) gain versus frequency curves are similar in the two bands and have peaks matching within 0.02 dB between the two bands.

We conclude, therefore, that while the array gain versus frequency curves are not truly periodic, most of the characteristics of the curves are maintained across widely different frequency ranges, including the spacings and approximate amplitudes of most of the peaks, as well as the approximate maximum array gain (to within 0.1 dB).

Array Gain Versus Bandwidth

The analysis of the preceding section looked only at two bandwidths: zero and 16.6 MHz. In order to simplify evaluation of results over a wide range of bandwidths, the amount of data associated with each bandwidth must be reduced. To that end, each center-frequency-versus-array gain curve will be reduced to two values: the maximum occurring across the 100 MHz analysis span and the median across that same span.

Figure 11 shows maximum array gain as a function of bandwidth for each of the 24 combinations of array configuration and cyclic delays (short or long). The plot includes 24 curves, though some are indistinguishable because they overlay one another.

The following general observations regarding the maximum array gains are apparent.
- As noted previously, at zero bandwidth the maximum narrowband array gain as a function of frequency is close to $10 \log(N_{ANT})$ dB in each of 24 cases (within 0.7 dB for the eight-antenna circular array cases and within 0.1 dB for the other 22 cases).
- As bandwidth increases from 0 MHz, the array gain drops more rapidly when transmissions employ long cyclic delays (the solid curves) rather than short cyclic delays (the dashed curves).
- The broadband array gain goes to ≤ 0.2 dB at multiples of 40 MHz bandwidth in all cases. Note that all cyclic delays are multiples of 25 ns (1/40 MHz).
- The broadband array gain goes to ≤ 0.2 dB at multiples of 20 MHz bandwidth in the following cases, all of which involve cyclic delays that are multiples of 50 ns (1/20 MHz).[21]

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[21] In fact, the broadband array gain goes to approximately zero dB at non-zero multiples of the reciprocal of the highest common submultiple of the cyclic delays:

   40 MHz for short cyclic delays with NANT ≥ 5, where the highest submultiple of the delays is 25 ns (Table 1);
   20 MHz for short cyclic delays with NANT = 4, where the highest submultiple of the delays is 50 ns (Table 1);
   5 MHz for short cyclic delays with NANT = 2, where the highest submultiple of the delays is 200 ns (Table 1);

◊   When long cyclic delays are transmitted on any of the 12 arrays; and
◊   When short cyclic delays are transmitted on arrays having no more than four antennas ($N_{ANT} \leq 4$).

Figure 12 is identical to Figure 11 except that it shows *median* array gain across the 100 MHz span as a function of bandwidth rather than the *maximum* array gain. The medians are generally somewhat lower than the maxima—particularly for the arrays with five or more antennas.

As a test of whether the results for the analyzed 5700 to 5800 MHz span are applicable more generally, a center frequency span from 2400 to 2500 MHz was analyzed for one selected array (square-1 with five antennas) with both short and long cyclic delays.[22] Figure 13 compares the maximum array gain versus bandwidth across the two frequency spans. The curves are nearly identical—matching within 0.1 dB. Figure 14 provides a similar comparison for median array gain across the two frequency spans. Broadband array gains are nearly identical, and narrowband array gains (the left end of the curves) differ by a maximum of 0.3 dB.

The results shown in Figure 11 and Figure 12 are discussed in more detail in the subsequent subsections of this report for the specific cases of bandwidths associated with:
* power spectral density measurements;
* the full bandwidth of a transmission; and
* the portion of transmitted bandwidth that falls within a given FCC rule band for channels that straddle the boundary between rule bands.

Array Gain Applicable to Power Spectral Density Limits

In applying rules that require a reduction in *power spectral density limits* based on directional gain, it is reasonable to base the reductions on the directional gain applicable to the measurement bandwidth that is specified for power spectral density limits in the applicable rule. With respect to rules under which IEEE 802.11 systems operate, the specified measurement bandwidth is 1 MHz except in section 15.247 of the rules, where a 3-kHz measurement bandwidth is specified.[23]

It was noted that in Figure 11 the maximum array gain at zero bandwidth is very close to $10 \log(N_{ANT})$ dB, where N_{ANT} is the number of transmit antennas. The array gain at 3-kHz bandwidth is essentially indistinguishable from the narrowband (*i.e.*, zero bandwidth) array gain. Consequently, where power spectral density is specified in 3-kHz bandwidth, it is appropriate to use the narrowband array gain, the maximum of which is given approximately by $10 \log(N_{ANT})$ dB. For 1 MHz bandwidth further analysis is required because of the steep dropoff of maximum array gain with bandwidth for curves involving long cyclic delays.

Figure 15 shows plots of both maximum (red curves) and median (blue curves) array gains in 1 MHz bandwidth as a function of number of transmit antennas.[24] The graph includes curves all twelve array configurations and both sets of cyclic delays bandwidth in addition to maximum and median results—a

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20 MHz for long cyclic delays with NANT = $\geq$ 5, where the highest submultiple of the delays is 50 ns (Table 2); 5 MHz for short cyclic delays with NANT = 4, where the highest submultiple of the delays is 200 ns (Table 2); 2.5 MHz for short cyclic delays with NANT = 2, where the highest submultiple of the delays is 400 ns (Table 2).

[22] The five-antenna square-1 array with short cyclic delays was chosen because it exhibits significant, complicated variations in array gain with frequency for both the narrowband and broadband (16.6 MHz) cases and because it exhibited the greatest RMS deviation from periodicity over the 5700-5800 MHz span, as discussed in footnote 17.

[23] 47 CFR 15.247(e) specifies power spectral density in 3 kHz bandwidth. As of this writing, FCC's Notice of Proposed Rulemaking FCC13-22 of February 20, 2013 proposes a reduction in measurement bandwidth for one of the U-NII bands from 1 MHz to possibly as low as 3 kHz.

[24] Again, the maximum and minimum are computed across a 100-MHz wide range of center frequencies (5700-5800 MHz).

total of 48 curves, though many overlay one another. The black line represents $10 \log(N_{ANT})$ dB, the theoretical maximum array gain for N_{ANT} transmit antennas. The dashed red curves are close to the black line, indicating that the maximum array gain for all twelve array configurations closely matches $10 \log(N_{ANT})$ dB when the transmissions have short cyclic delays. The maximum array gain in 1 MHz bandwidth is lower with long cyclic delays (solid red curves) than with short cyclic delays (dashed red curves) by amounts ranging from 0.6 to 1.3 dB due to the steeper falloff of array gain width increasing bandwidth that was observed for long cyclic delays in the solid curves in Figure 11. Median array gains closely match the maxima for two- and four-antenna arrays, but the median curves tend to flatten for larger numbers of antennas.

We judge that the use of $10 \log(N_{ANT})$ dB is appropriate for calculating array gain when computing required reductions in power spectral density in 1 MHz or narrower bandwidths for the following reasons.
- Because some transmissions occur using the short cyclic delays it is not appropriate to assume the roughly 1 dB reduction in maximum gain that occurs when using long cyclic delays.
- The use of maximum rather than median gains is appropriate given that the power-spectral-density limit must be achieved at all frequencies across the transmission spectrum and that near-maximum array gain is likely to occur in many or perhaps most channels given that at least one to two such array gain peaks occur in each 40 MHz span of center frequencies (*e.g.*, Figure 6 through Figure 9).

Array Gain Applicable to Power Measurements Across the Full Bandwidth of a Transmission

As noted earlier, the FCC rules often require a reduction of *transmit power* based on directional gain. In applying those rules, it is reasonable to compute the power reduction based on the *broadband* directional gain across the portion of a transmitted signal that falls within the rule band under consideration. Except where a transmitted channel is split across rule bands, this means that we are interested in the directional gain applicable to the entire bandwidth of signal transmitted on a channel. Since array gain is one component of the total directional gain, we want to know the broadband array gain for the transmitted signal as a whole. For signals with rectangular spectra such as the OFDM transmissions used by modern IEEE 802.11 devices,[25] this means that we want to know the array gain corresponding to beam patterns that have been averaged across the rectangular portion of the transmission bandwidth.

The bandwidths of IEEE 802.11 OFDM transmissions are shown as vertical black lines in Figure 11 and Figure 12.[26] We note the following:
- Maximum broadband array gains across the bandwidths of transmissions in 40 and 80 MHz channels are less than 0.5 dB. Based on the pattern of broadband array gain going to approximately 0 dB at multiples of 40 MHz of bandwidth, one can expect the same to be true for transmissions in 160 MHz channels.
- Maximum broadband array gains for the bandwidths of transmissions in 20 MHz channels (802.11a/g or n) are as follows:
  ◊ < 0.8 dB for arrays with any N_{ANT} when transmitting with long cyclic delays.
  ◊ ≤ 0.8 dB for arrays with $N_{ANT} \le 4$ when transmitting short cyclic delays. [27]

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[25] OFDM is used for all transmissions except for modes designed for backward compatibility with 802.11b and earlier devices, which use direct sequence spread spectrum (DSSS) signals.

[26] IEEE 802.11 OFDM transmissions have subcarriers at intervals of 312.5 kHz. The bandwidths shown correspond to the spacing between the centers of the outermost carriers plus 312.5 kHz to account for a modulation width of 312.5 kHz/2 on each side of the outer carriers. The resulting bandwidths are 16.5625 MHz, 17.8125 MHz, 36.5625 MHz, 76.5625 MHz, and 156.5625 MHz, respectively for channel widths of 20 MHz (802.11a/g), 20 MHz (802.11n High Throughput), 40 MHz (802.11n), 80 MHz (802.11ac), and 160 MHz (802.11ac).

[27] The gains are near zero because gains go to ≈ 0 dB at multiples of 20 MHz when cyclic delays are multiples of 50 ns, as occurs with short cyclic delays when $N_{ANT} \le 4$ (see Table 1) and with long delays regardless of N_{ANT} (see Table 2).

◊ 2.7 to 3.5 dB for arrays with $N_{ANT} \geq 5$ when transmitting with short cyclic delays. (The median broadband array gains for this group range from 2.1 to 3.0 dB.)

Figure 16 shows plots of both maximum (red curves) and median (blue curves) array gains in the bandwidths associated with 20 MHz 802.11 channels as a function of number of transmit antennas.[28] The plotted values are an average (in decibels) of the gains computed for the transmit bandwidths used by 802.11a/g modes (16.6 MHz) and 802.11n High Throughput (HT) modes (17.8 MHz) when operating in 20 MHz channels. The graph includes curves for all twelve array configurations and both sets of cyclic delays bandwidth in addition to maximum and median results—a total of 48 curves, though many overlay one another.

For four or fewer antennas, the maximum array gain is well under 1 dB. We propose to omit the array gain from the directional gain calculations in that case (*i.e.*, we will assume that array gain = 0 dB).

For five or more antennas, both the maximum and median array gains are about 3 dB for short cyclic delays and well under 1 dB for long cyclic delays. Because both short and long cyclic delays are used by 802.11 devices—sometimes within the same transmission, we propose to assume a constant array gain of 3 dB for cyclic delay diversity transmissions on five or more antennas.

Thus, the solid black line in Figure 16 represents the proposed guidance for computing array gain for 802.11 devices when transmitting cyclic delay diversity signals in 20 MHz channels. For 802.11 cyclic delay diversity transmissions in channel widths of 40 MHz and wider, we propose to assume that array gain is 0 dB.

Array Gain Applicable to Power Measurements on Channels That Straddle the Boundary Between Rule Bands

For operations in the 5 GHz bands, some IEEE 802.11 channels straddle the boundary between two FCC rule bands. Figure 17 illustrates the applicable FCC rule bands along with the channel plan for the 5 GHz bands under the IEEE 802.11n standard and a draft version of IEEE 802.11ac. The four rows in the illustration correspond to 20, 40, 80, and 160 MHz channel widths. Channels that straddle FCC band boundaries are outlined in blue.[29]

For band-straddling channels, the power in each band is limited by the rules for that band, and it is appropriate that required power reductions be computed based on the broadband directional gain for the portion of the transmit spectrum that falls in each band. Thus, in principle, we are interested in the broadband array gain across a bandwidth corresponding to the segment of the signal in each rule band.

Figure 17 identifies the bandwidths of left and right segments of each channel that straddles a band boundary as well as the maximum array gains associated with that bandwidth. The maximum array gains are derived from the data plotted in Figure 18, which is a repeat of the earlier plot of maximum array gain versus bandwidth (Figure 11) but with vertical dashed lines added to illustrate the bandwidths of the segments of transmissions that fall within each band for the channels that straddle the 5.725 MHz boundary, *i.e.*, the channels shown in red in Figure 17.

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[28] Again, the maximum and minimum are computed across a 100-MHz wide range of center frequencies (5700-5800 MHz).

[29] Though channel 165 is centered on the upper edge of the U-NII 3 band as defined at the time of this writing, it does not operate across the boundary between two rule bands because, under current rules and guidance, this channel must operate under section 15.247 of the rules. If the upper edge of the U-NII 3 band is extended to 5.850 GHz and non-frequency-hopping operations in the 5.725 to 5.850 GHz band under section 15.247 are prohibited, as proposed in FCC's Notice of Proposed Rulemaking FCC13-22 of February 20, 2013, the channel would then operate entirely within the extended U-NII 3 band.

As discussed previously, we chose to ignore the full-channel-width array gains for channels of 40 MHz or wider because the array gains were less than 1 dB. Likewise, the array gains of straddle-channel segments that are less than 1 dB will also be neglected. This includes both halves of the 160 MHz channel that is split across the band boundary at 5.25 GHz and the left-side segments of the 40 and 80 MHz channels that straddle the boundary at 5.725 GHz.

The two cases leading to broadband array gains greater than 1 dB require further discussion.
(1) The segment of the 20 MHz channel that falls to the left of the 5.725 GHz band boundary has a width of either 13.3 or 13.9 MHz depending on whether it is an 802.11a or 802.11n/ac (HT or VHT) transmission. The resulting maximum broadband array gain is 3.6 to 4.0 dB when short cyclic delays are transmitted from arrays with five or more antennas and between 0.2 and 1.6 dB in other cases. Note that our guidance for broadband array gain across the full width of a 20 MHz channel is to assume array gain = 0 dB when $N_{ANT} \leq 4$ and array gain = 3 dB when $N_{ANT} \geq 5$. Thus, the actual array gain for the partial channel that falls below the 5.725 GHz band boundary could exceed the full channel guidance by up to 1.6 dB when $N_{ANT} \leq 4$ and up to 1.0 dB when $N_{ANT} \geq 5$.
(2) The segments of the 20, 40, or 80 MHz channels that fall to the right of the boundary have bandwidths of 3.3 MHz except for 20 MHz 802.11n or ac (HT or VHT) transmissions, which extend 3.9 MHz above the 5.725 GHz band boundary. The resulting gains are ≤ 1.4 dB for $N_{ANT} = 2$ and $\leq 9.1 \log(N_{ANT})$ for $N_{ANT} > 2$ (e.g., up to 8.2 dB for eight antennas), whereas the guidance for full channel array gain assumes that array gain = 3 dB for 20 MHz channels when $N_{ANT} > 5$ and 0 dB in all other cases.

Because 802.11 testing is already enormously complex,[30] we will investigate below whether we can reasonably avoid requiring separate broadband array gain calculations for these two remaining straddle-channel cases. Before proceeding, it will be useful to understand the relationship between power spectral density limits and power limits in the U-NII rules. Section 15.407(a)(2) of the FCC rules specifies a power limit of "250 mW or 11 dBm + 10 log B, where B is the 26 dB emission bandwidth in MHz" and a power spectral density limit of "11 dBm in any 1 megahertz band" for the U-NII 2C band prior to reductions based on directional gain. If B < 20 MHz, then the bandwidth-dependent power limit (11 dBm + 10 log B) will apply. Consider a span of OFDM carriers that having a total effective width of B_{OFDM} (in MHz).[31] The emission bandwidth B will be greater than B_{OFDM} because B is measured at the 26 dB down points. A limit on power spectral density of PSD_{LIMIT} (in dBm per MHz) will effectively limit the power to $PSD_{LIMIT} + 10 \log(B_{OFDM}) + 1.2$ dB with the 1.2 dB term being related to the relationship between the specified measurement bandwidth and equivalent noise bandwidth of the measurement filter.[32] The power limit specified in the rules is 11 dBm + 10 log B, but since $B \geq B_{OFDM}$,

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[30] 802.11ac devices can operate in nine OFDM modulation modes on up to five channel bandwidths (20 MHz, 20 MHz HT, 40 MHz, 80 MHz, 160 MHz) in up to five bands. For each of these, MIMO operation can occur with various combinations of spatial multiplexing, cyclic delay diversity, and beamforming.

[31] IEEE 802.11 OFDM transmissions have subcarriers at intervals of 312.5 kHz. We can define B_{OFDM} as the spacing between the centers of the outermost carriers plus 312.5 kHz to account for a modulation width of 312.5 kHz/2 on each side of the outer carriers. For 20 MHz channels $B_{OFDM} = 16.6$ MHz for 802.11a or g transmissions and 17.8 MHz for 802.11n or ac (HT or VHT) transmissions.

[32] If each 1 MHz of the signal contains a power level of no more than PSD_{LIMIT} and if the signal extends over a bandwidth of B_{OFDM} (in MHz), then one would expect that the total power of the signal could not exceed $PSD_{LIMIT} + 10 \log(B_{OFDM})$. (We assume here that the amount of power contained beyond the B_{OFDM} edge is negligible.) However, the FCC permits power spectral density measurements to be performed using an instrument with a measurement filter that is 1 MHz wide at the 6 dB down points (ANSI C63.2). For a typical modern spectrum analyzer with Gaussian-shaped filters, the 6-dB bandwidth is larger than the 3-dB bandwidth by a factor of 1.414 ["User's and Programmer's Reference, Volume 1, Core Spectrum Analyzer Functions, PSA Series Spectrum Analyzers", Agilent Technologies, June 2008, section 2.3.1, p.67] or 1.415 [Christoph Rauscher, "Fundamentals of Spectrum Analysis", Rohde & Schwarz, 2001, p.48] and the equivalent noise bandwidth is larger than the 3 dB bandwidth by a factor of 1.055 ["Specifications Guide, Agilent Technologies PSA Series Spectrum Analyzers", Agilent Technologies, Dec 2006, p.81, Note a] or 1.065 [Rauscher, p.48]. Thus, a measurement filter having a 6 dB

the power limit is greater than 11 dBm + 10 log(B_{OFDM}). Consequently, the limits prior to adjustment for directional gain are such that, satisfying the power spectral density limit falls no more than 1.2 dB short of ensuring compliance with the power limit. (The other U-NII bands have power limits and power spectral density limits that are higher or lower than those of U-NII 2C, but the relationship between the power and power spectral density limits remains the same.)

Furthermore, if adjustments in the limits are required due to directional gain exceeding 6 dBi, the downward adjustment in the power spectral density limit due to narrowband directional gain will always be greater than the downward adjustment in the power limit due to broadband directional gain because narrowband directional gain always exceeds broadband directional gain. Thus, the adjustments go in the direction of further ensuring compliance with the power limit.

Consequently, we can conclude that compliance with the power spectral density limit, as adjusted for narrowband directional gain, ensures that power of the signal segment in a given band cannot exceed the power limit, adjusted for broadband directional gain, by more than 1.2 dB. In practice, the 1.2 dB factor will be reduced by all of the following:
- 10 log(B/ B_{OFDM}), because the 26 dB bandwidth B is greater than the effective bandwidth of the OFDM signal;
- Any ripple or non-flatness in the OFDM signal spectrum;
- The difference between narrowband array gain and broadband array gain if total narrowband directional gain (antenna gain + array gain) exceeds 6 dBi.

Case (1) – Left Segment of 20-MHz Channel Straddling 5.725 GHz. In case (1), the segment of the 20 MHz channel that falls to the left of the 5.725 GHz band boundary has a width of either 13.3 or 13.9 MHz. The maximum broadband array gain is 4.0 dB when short cyclic delays are transmitted from arrays with $N_{ANT} \geq 5$ and 1.6 dB in all other cases. We already will require an assumption of 3 dB broadband array gain in the former case based on proposed guidance for channels that are not split between bands. In addition, the considerations of the preceding paragraph suggest that compliance with the power limit as adjusted for broadband directional gain, will likely be ensured by compliance with the power spectral density limit, as adjusted for narrowband array gain.[33]

Case (2) – Right Segment of 20, 40, or 80 MHz Channel Straddling 5.725 GHz. In case (2) the segment of the 20 MHz channel that falls to the right of the 5.725 GHz band boundary has a width of either 3.3 MHz or 3.9 MHz. This can result in broadband gains as high as 1.4 dB for $N_{ANT} = 2$ and ≤ 9.1 log(N_{ANT}) for $N_{ANT} > 2$ (e.g., up to 8.2 dB for eight antennas), so the difference between narrowband and broadband gain may be relatively small. However, the U-NII 3 band has 6 dB higher limits on both power and power spectral density than the U-NII 2C band;[34] consequently, compliance with the U-NII 3 limits is not expected to be a problem unless the power of the OFDM subcarriers above the band boundary are raised substantially above those below the band boundary for a channel that crosses the boundary. We do not anticipate such large imbalances in subcarrier amplitudes to occur within a single OFDM channel.

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bandwidth of 1 MHz would have an equivalent noise bandwidth of 1.055/1.414 = 0.746 MHz or 1.065/1.415 = 0.753 MHz. For a signal with a flat spectrum, the power in each 1 MHz could be higher than PSD_{LIMIT} (which is specified based on a filter with an effective bandwidth of the about 0.75 MHz) by a factor of
10 log(1 MHz/0.75 MHz) dB = 1.2 dB.

[33] Even apart from the bandwidth ratio (B/B_{OFDM}) and the spectral ripple effects, the directional gain considerations alone will ensure compliance for cases when directional antenna gain + 10 log(N_{ANT}) ≥ 7.2 dB because the narrowband directional gain exceeds the broadband directional gain by more than 1.2 dB in every case.

[34] 15.407(a)(2) and (a)(3). Note also that, a rulemaking initiated by the FCC's Notice of Proposed Rulemaking FCC13-22 of February 20, 2013 is open at the time of this writing. That rulemaking could eliminate the bandwidth dependence of the power limit in the U-NII 3 band, leaving a fixed 1 watt power limit (for up to 6 dBi gain) that would be even more difficult to exceed.

We conclude, therefore, that for the IEEE 802.11 channel assignments that exist at the time of this report, separate computation of broadband array gain for the individual segments of a band-straddling channel is not necessary to ensure compliance with FCC power limits.

CONCLUSIONS

The analysis contained in this report leads to the following recommendations for performing directional gain calculations of IEEE 802.11 transmitters when employing cyclic delay diversity signals without the use of spatial multiplexing or of intentional beamforming.

Total directional gain (dBi) = gain of individual transmit antennas (dBi) + array gain (dB)

When determining reductions in power spectral density limits, array gain is calculated as follows:

Array gain = $10 \log(N_{ANT})$, where N_{ANT} is the number of transmit antennas.

When determining reductions in conducted power limits, array gain is calculated as follows:

Array Gain = 0 dB for $N_{ANT} \leq 4$;
Array Gain = 0 dB for channel widths $\geq$ 40 MHz for any N_{ANT};
Array Gain = 3 dB for 20-MHz channel widths with $N_{ANT} \geq 5$.

The guidance presented here is expanded in Knowledge Database (KDB) Publication 662911, "Emissions Testing of Transmitters with Multiple Outputs in the Same Band (e.g., MIMO, Smart Antenna, etc)", to include cases involving multiple spatial streams. The guidance presented here is also subject to change. The latest version of KDB Publication 662911 or other relevant KDB publications should be consulted for up to date guidance.
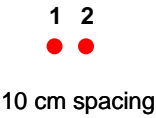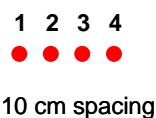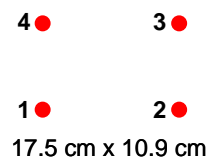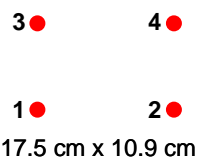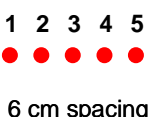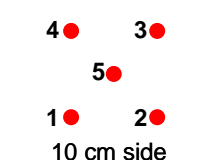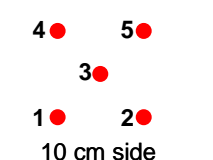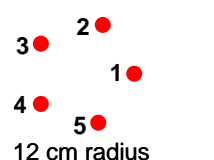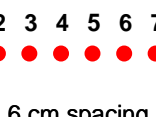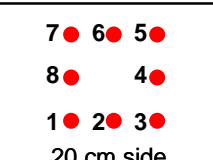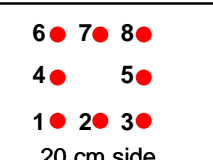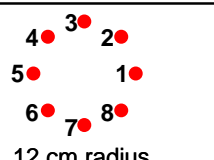
N_{ANT}	LINE	RECTANGLE 1 or SQUARE 1	RECTANGLE 2 or SQUARE 2	CIRCLE
2	**1 2** ● ● 10 cm spacing			
4	**1 2 3 4** ● ● ● ● 10 cm spacing	**4**● **3**● **1**● **2**● 17.5 cm x 10.9 cm	**3**● **4**● **1**● **2**● 17.5 cm x 10.9 cm	
5	**1 2 3 4 5** ● ● ● ● ● 6 cm spacing	**4**● **3**● **5**● **1**● **2**● 10 cm side	**4**● **5**● **3**● **1**● **2**● 10 cm side	**2**● **3**● **1**● **4**● **5**● 12 cm radius
8	**1 2 3 4 5 6 7 8** ● ● ● ● ● ● ● ● 6 cm spacing	**7**● **6**● **5**● **8**● **4**● **1**● **2**● **3**● 20 cm side	**6**● **7**● **8**● **4**● **5**● **1**● **2**● **3**● 20 cm side	**4**● **3**● **2**● **5**● **1**● **6**● **7** **8**● 12 cm radius

Figure 1. Array Configurations for FCC Analysis of 802.11 Cyclic Delay Diversity
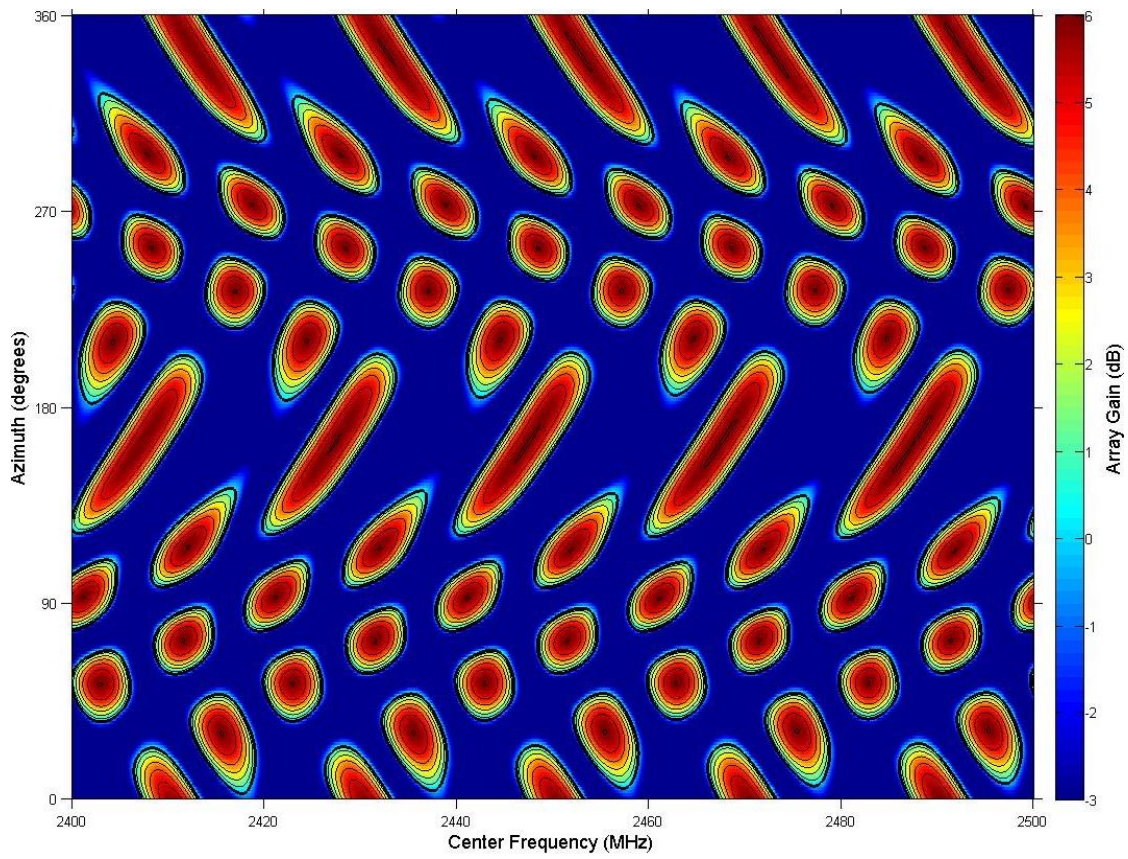
(a) Short Cyclic Delays (Table 1)
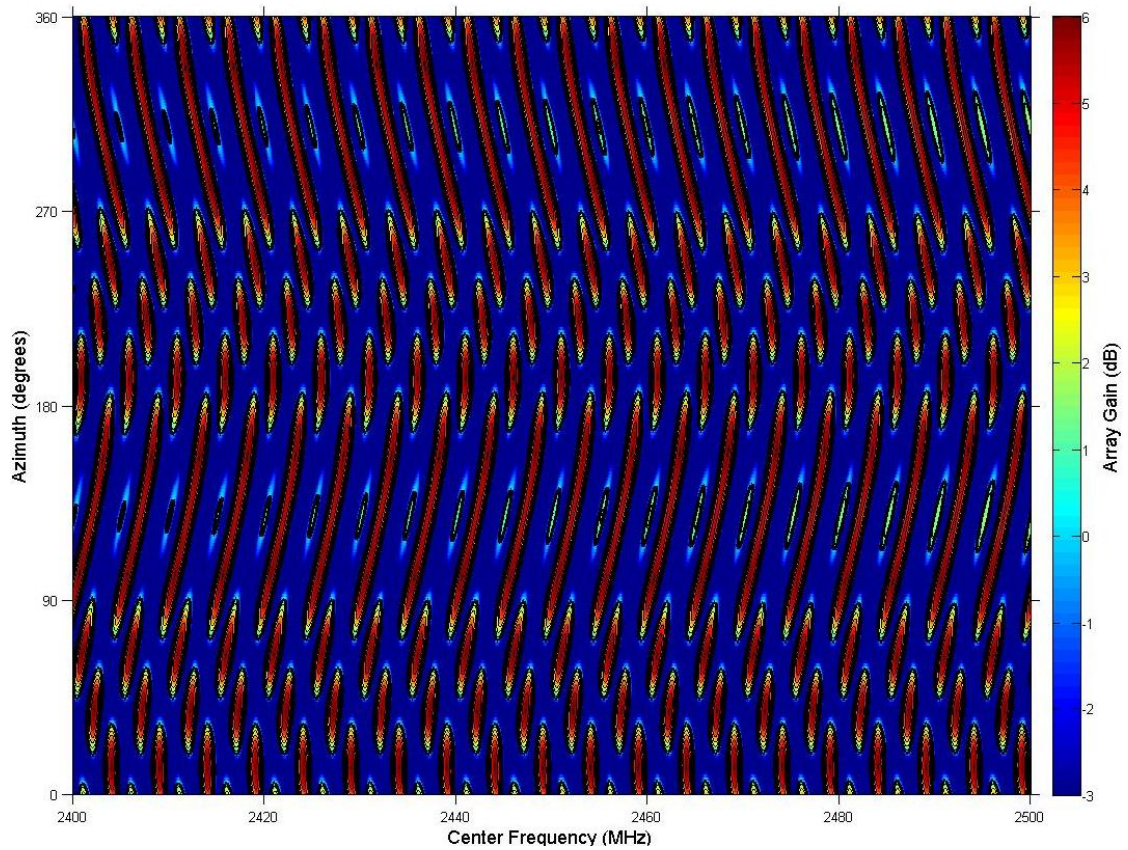
(b) Long Cyclic Delays (Table 2)

Figure 2. Narrowband Azimuth Beam Patterns (2400-2500 MHz) of Four-Antenna Rectangle-2

17
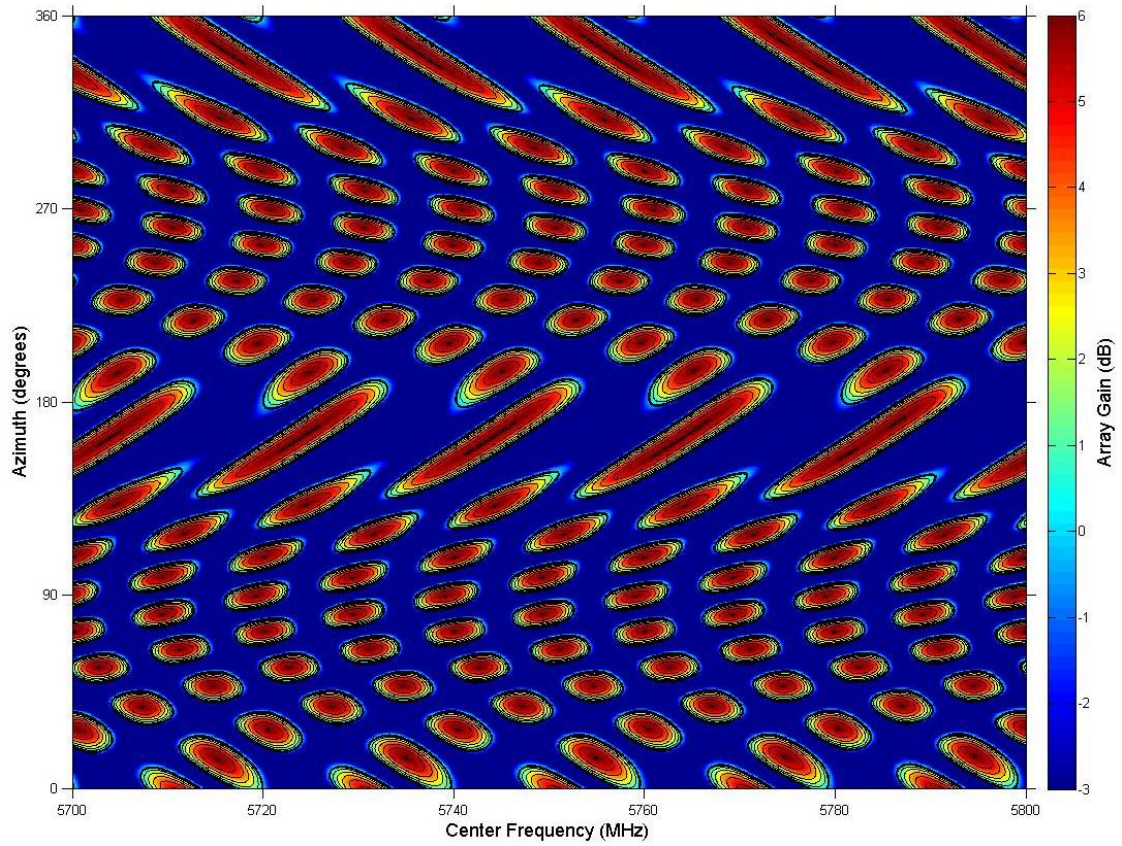
(a) Short Cyclic Delays (Table 1)
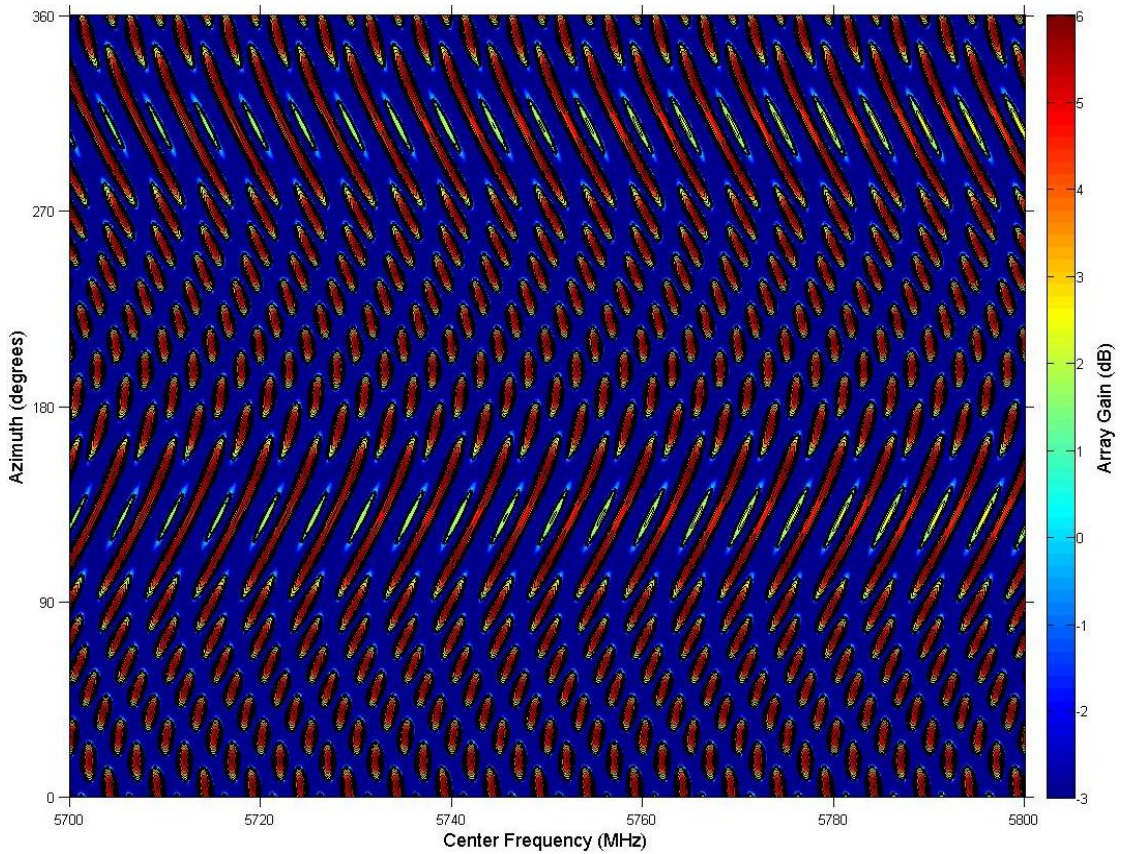
(b) Long Cyclic Delays (Table 2)

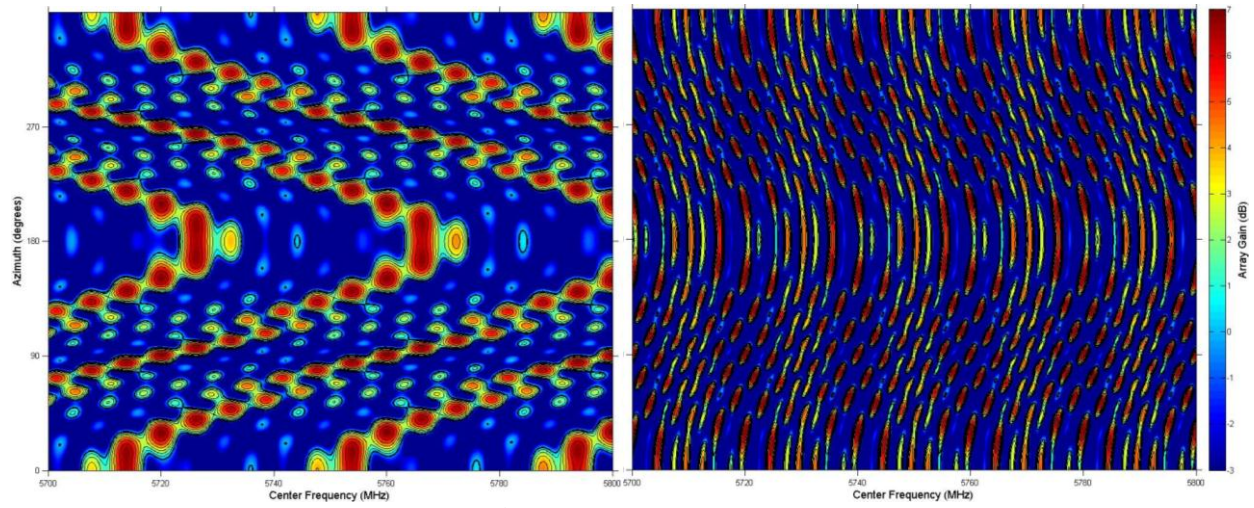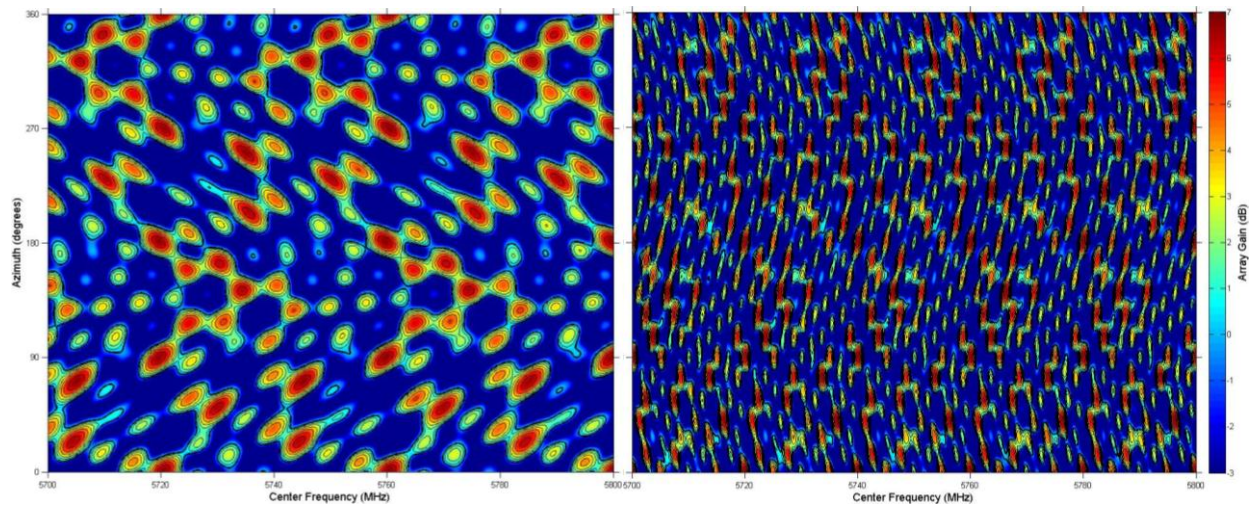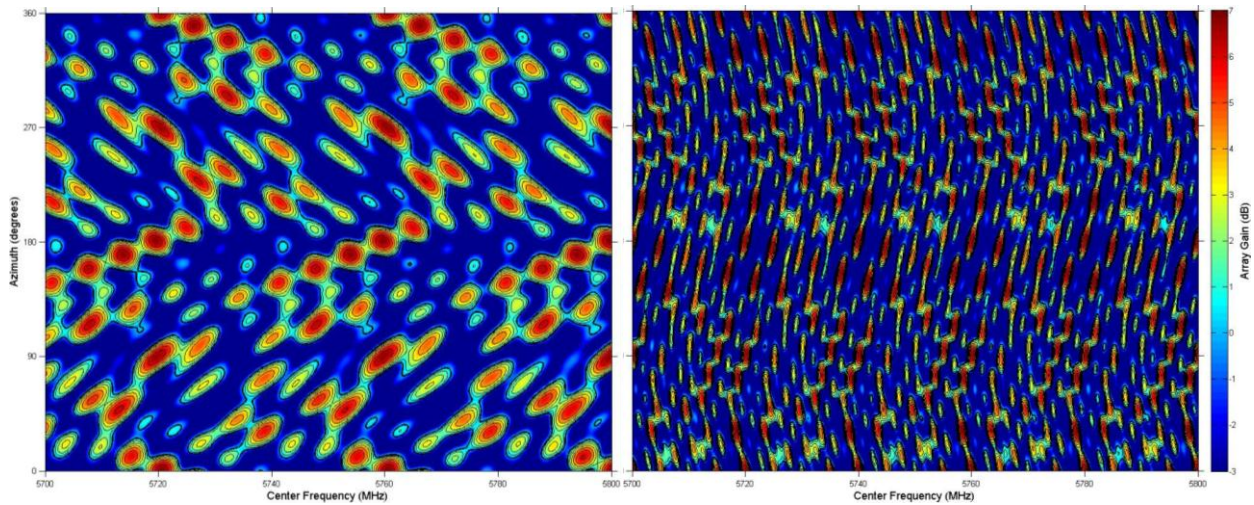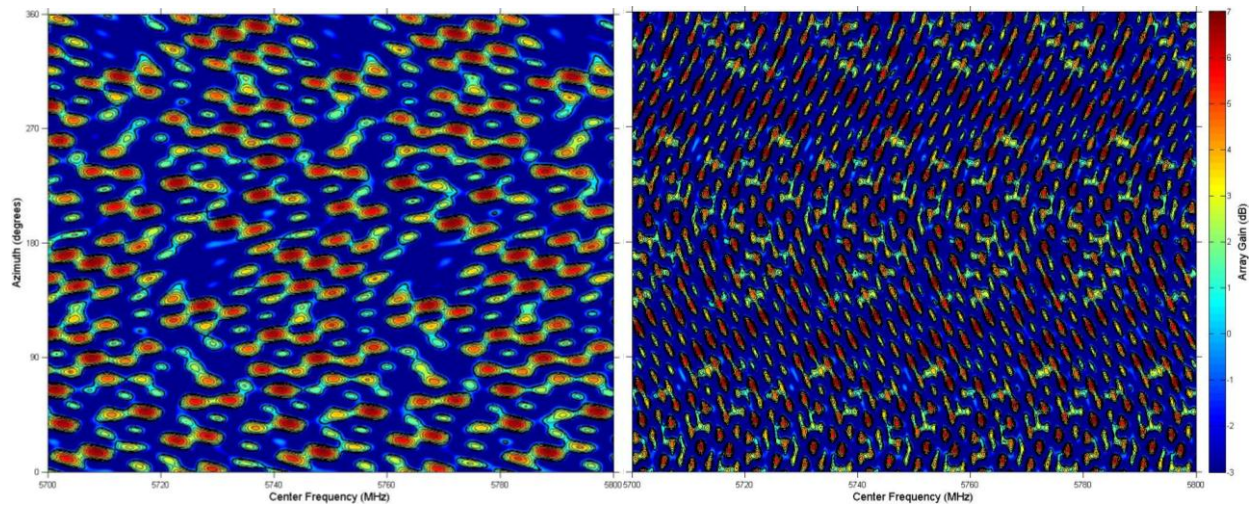*Figure 3. Narrowband Azimuth Beam Patterns (5700-5800 MHz) of Four-Antenna Rectangle-2*
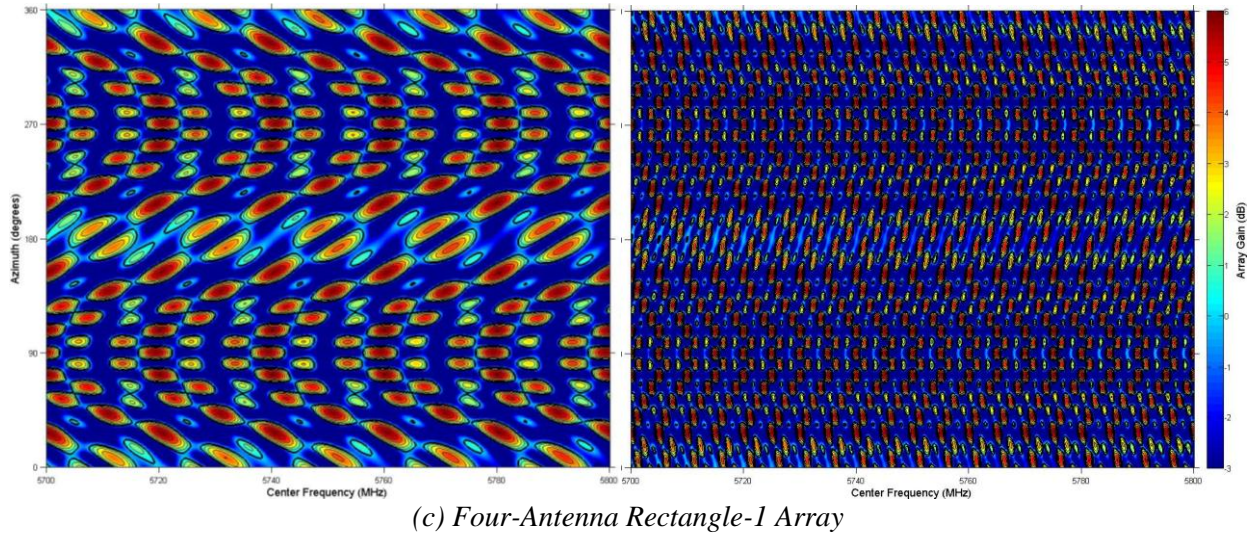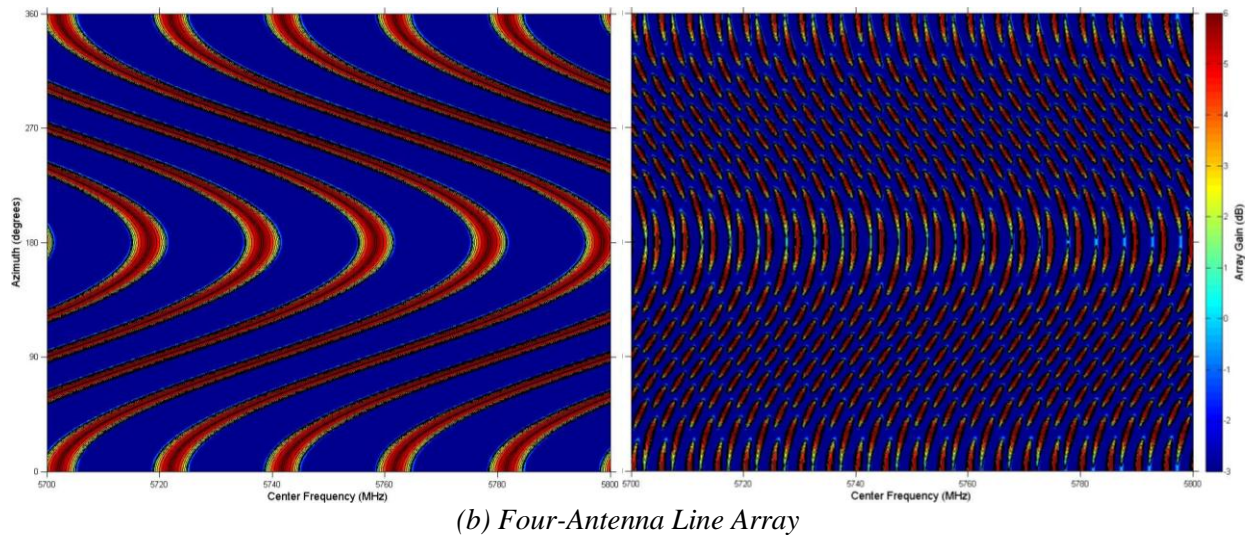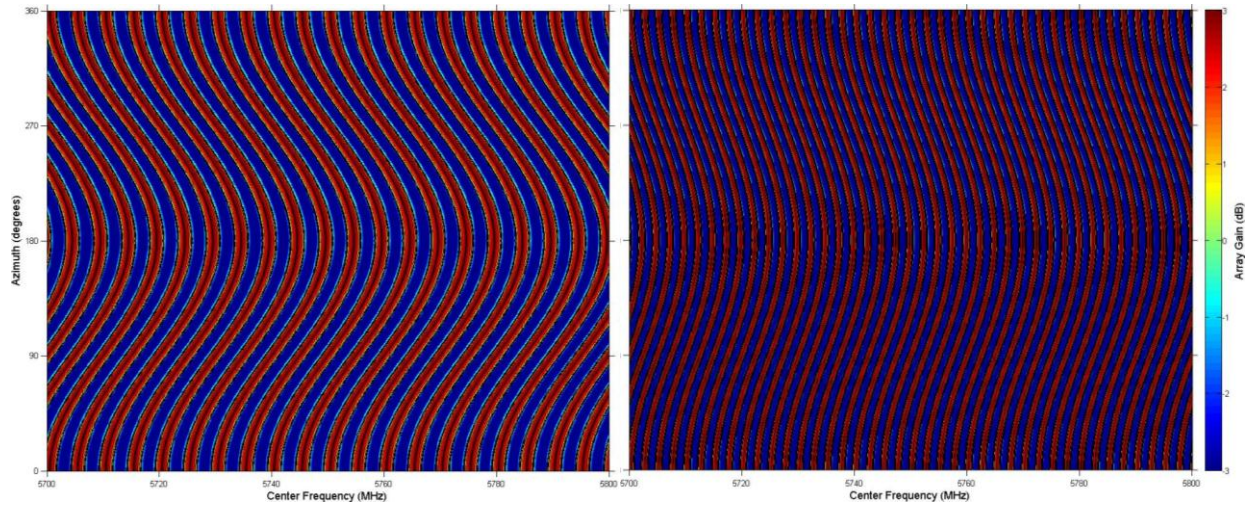
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(a) Two-Antenna Line Array

(b) Four-Antenna Line Array

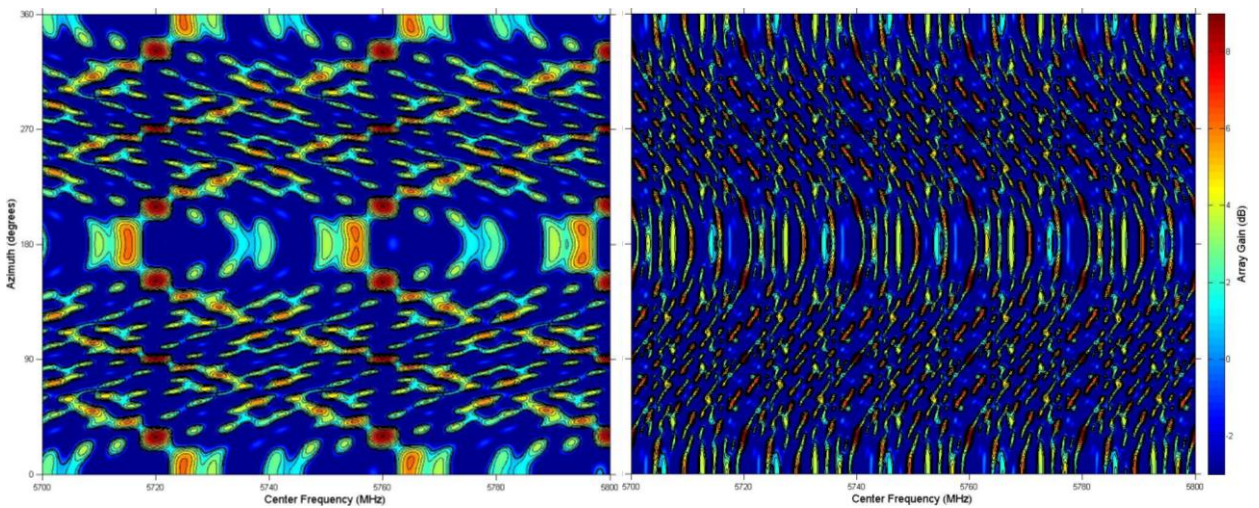*(c) Four-Antenna Rectangle-1 Array*

Short Cyclic Delays (Table 1)   *Long Cyclic Delays (Table 2)*

Figure 4. Narrowband Azimuth Beam Patterns (5700-5800 MHz) for Eleven Array Configurations

(continued next page)

(d) 5-Antenna Line Array

(e) 5-Antenna Square-1 Array

(f) 5-Antenna Square-2 Array

Short Cyclic Delays (Table 1)   Long Cyclic Delays (Table 2)
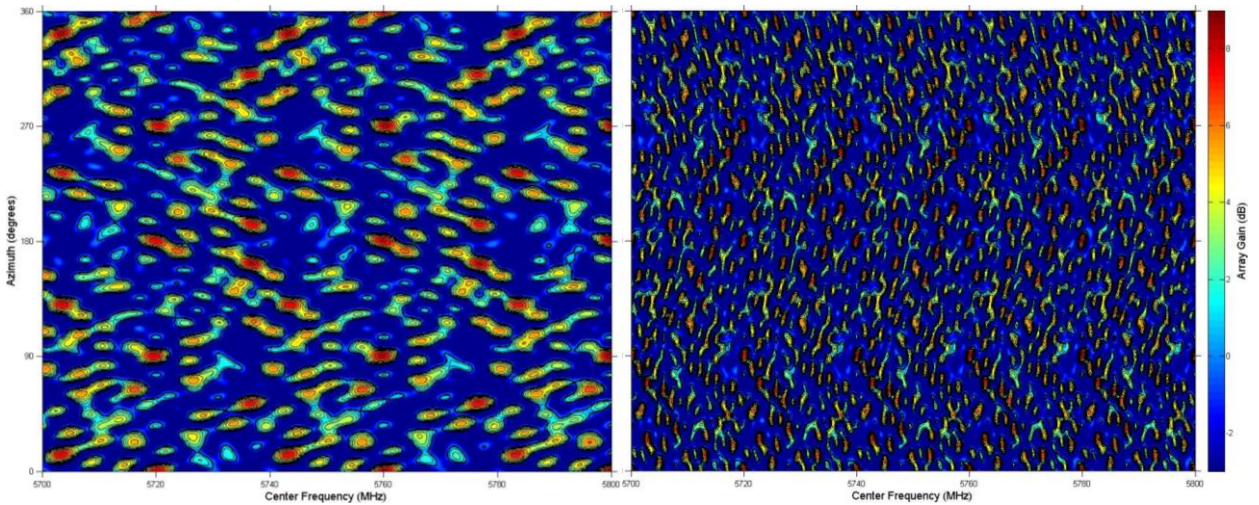
Figure 4. Narrowband Azimuth Beam Patterns (5700-5800 MHz) for Eleven Array Configurations (continued)

(g) 5-Antenna Circle Array
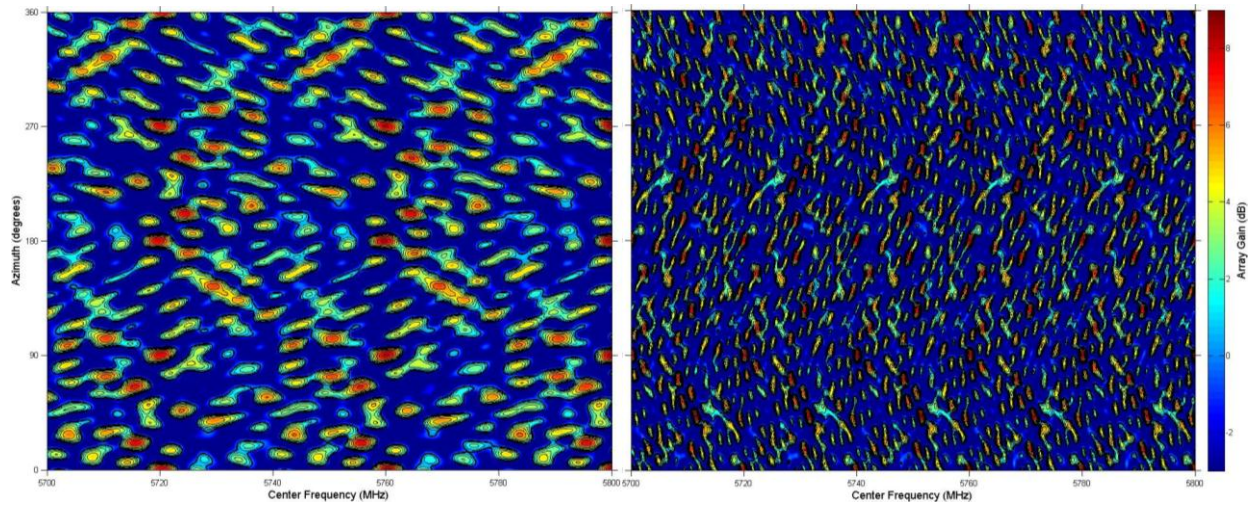

(h) 8-Antenna Line Array
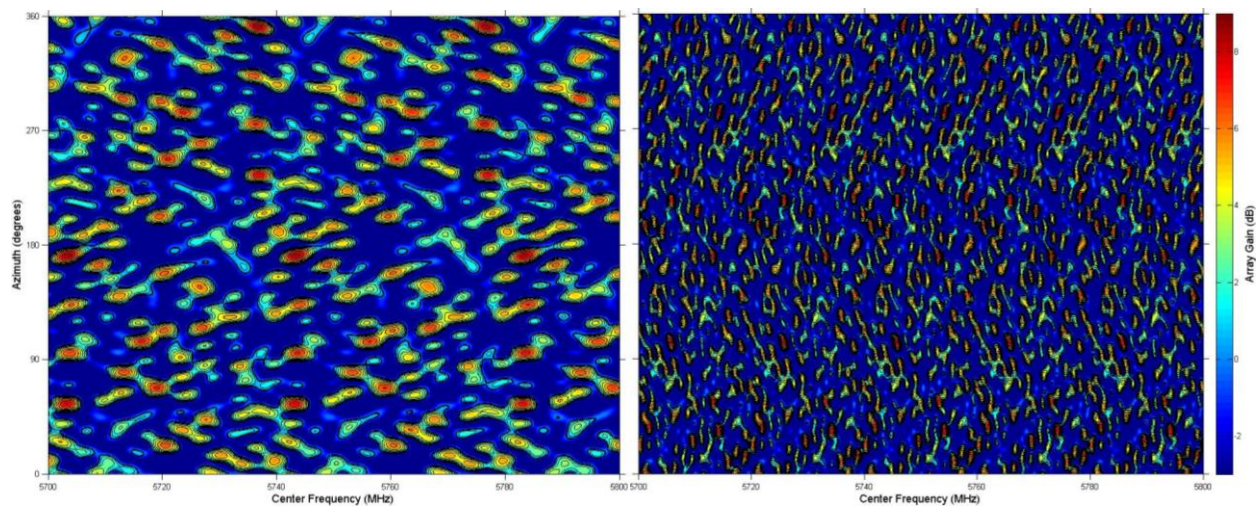

8-Antenna Square-1

Short Cyclic Delays (Table 1)   *Long Cyclic Delays (Table 2)*

Figure 4. Narrowband Azimuth Beam Patterns (5700-5800 MHz) for Eleven Array Configurations
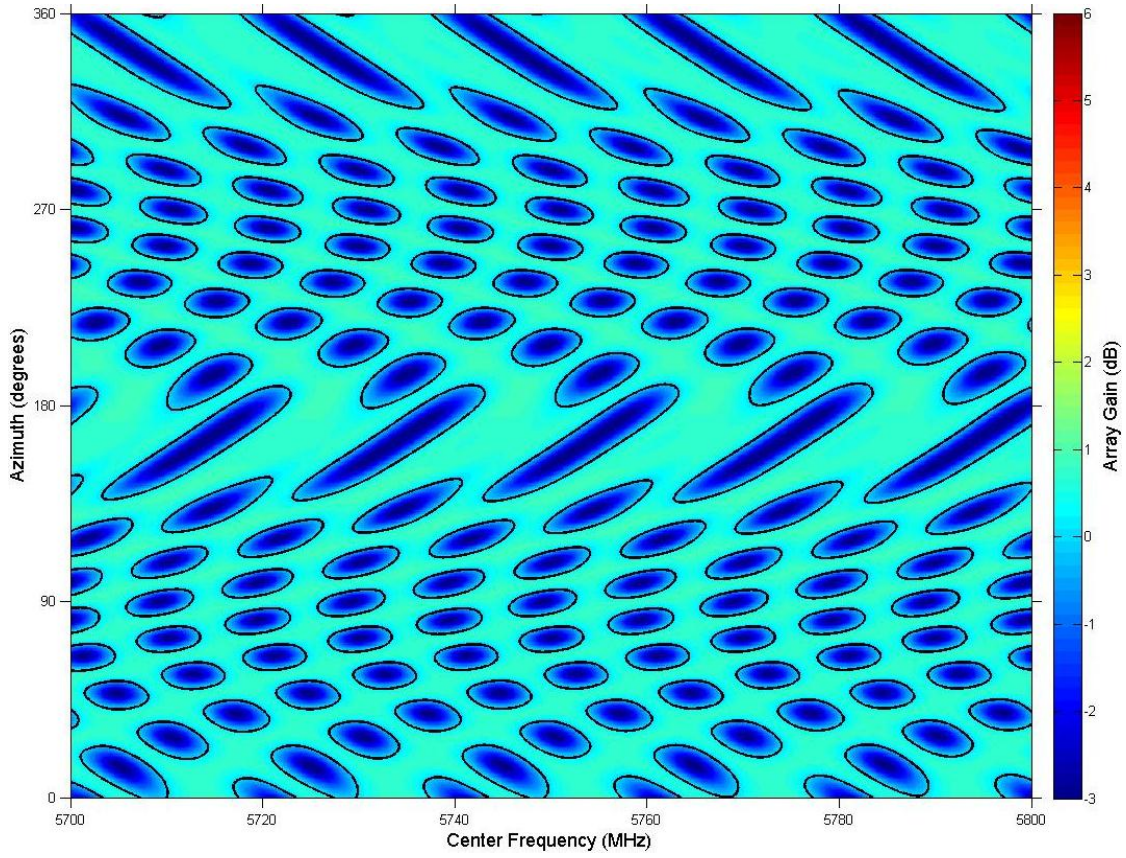*(continued)*

(i) 8-Antenna Square-2
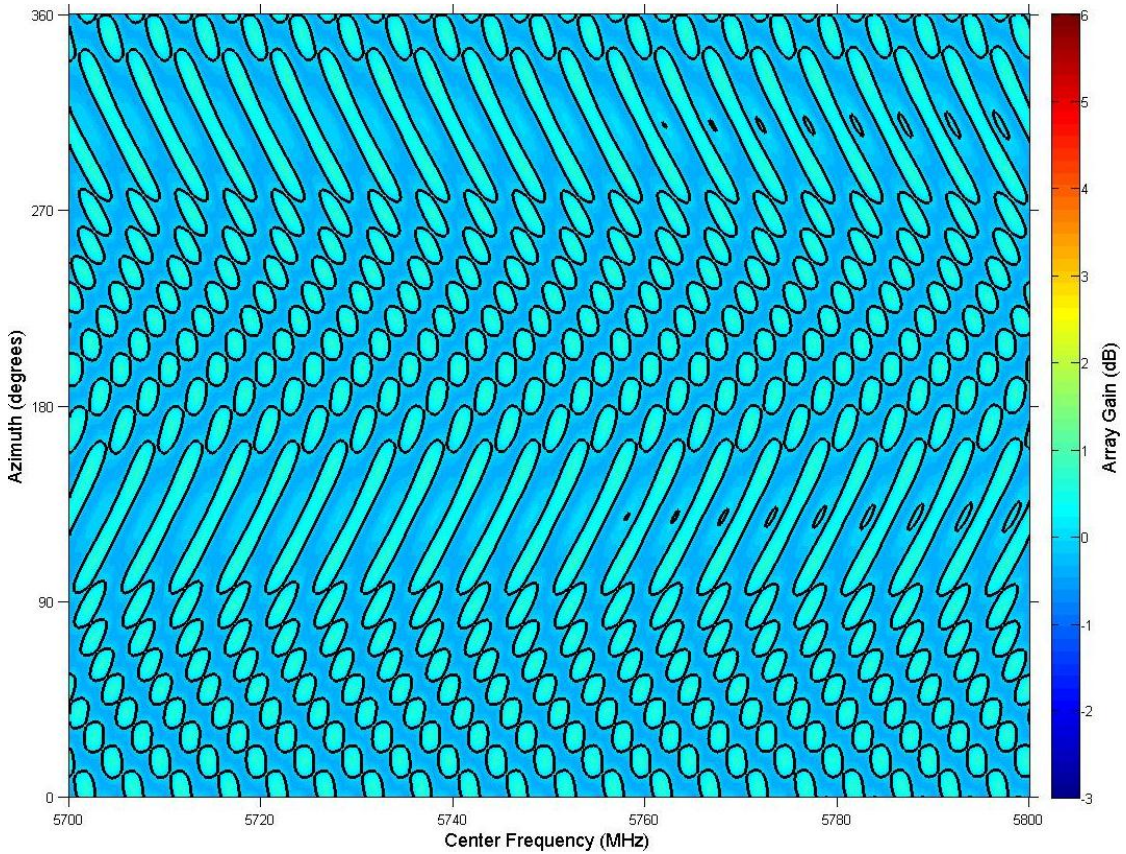

(j) 8-Antenna Circle Array

Short Cyclic Delays (Table 1)   *Long Cyclic Delays (Table 2)*

Figure 4. Narrowband Azimuth Beam Patterns (5700-5800 MHz) for Eleven Array Configurations (continued)
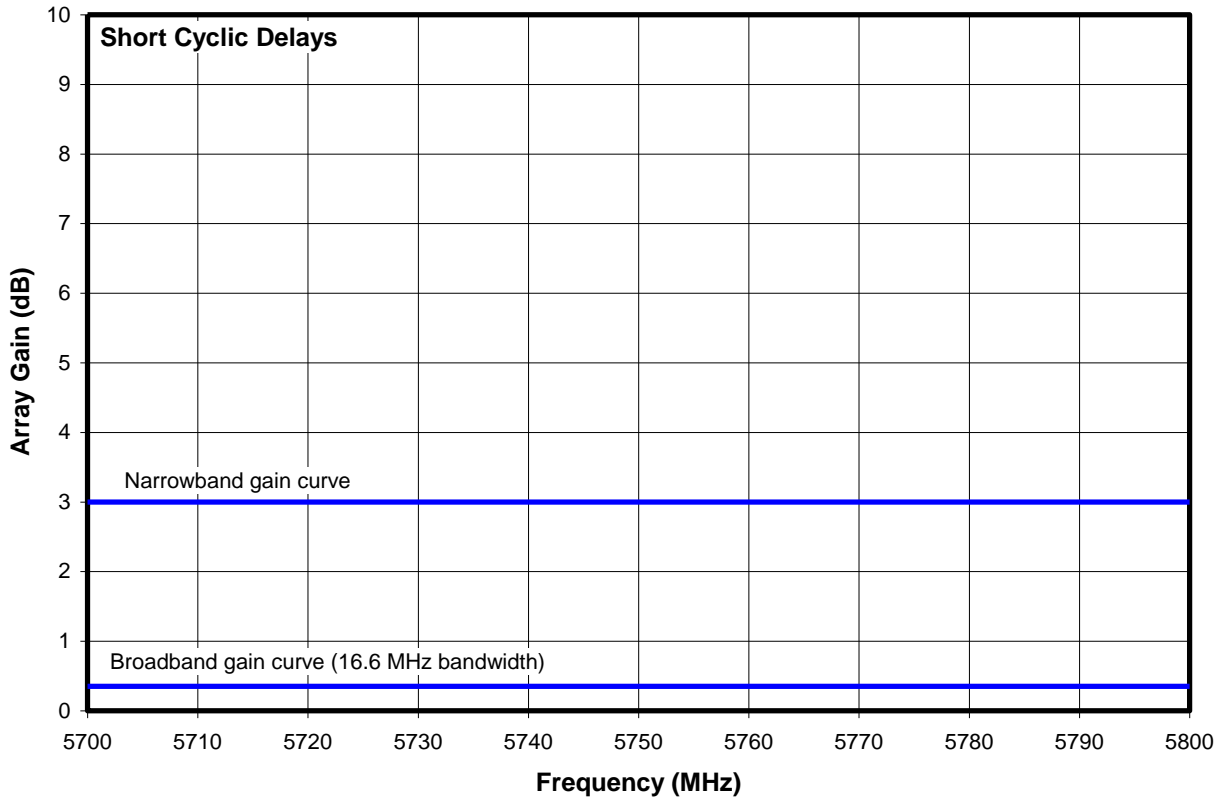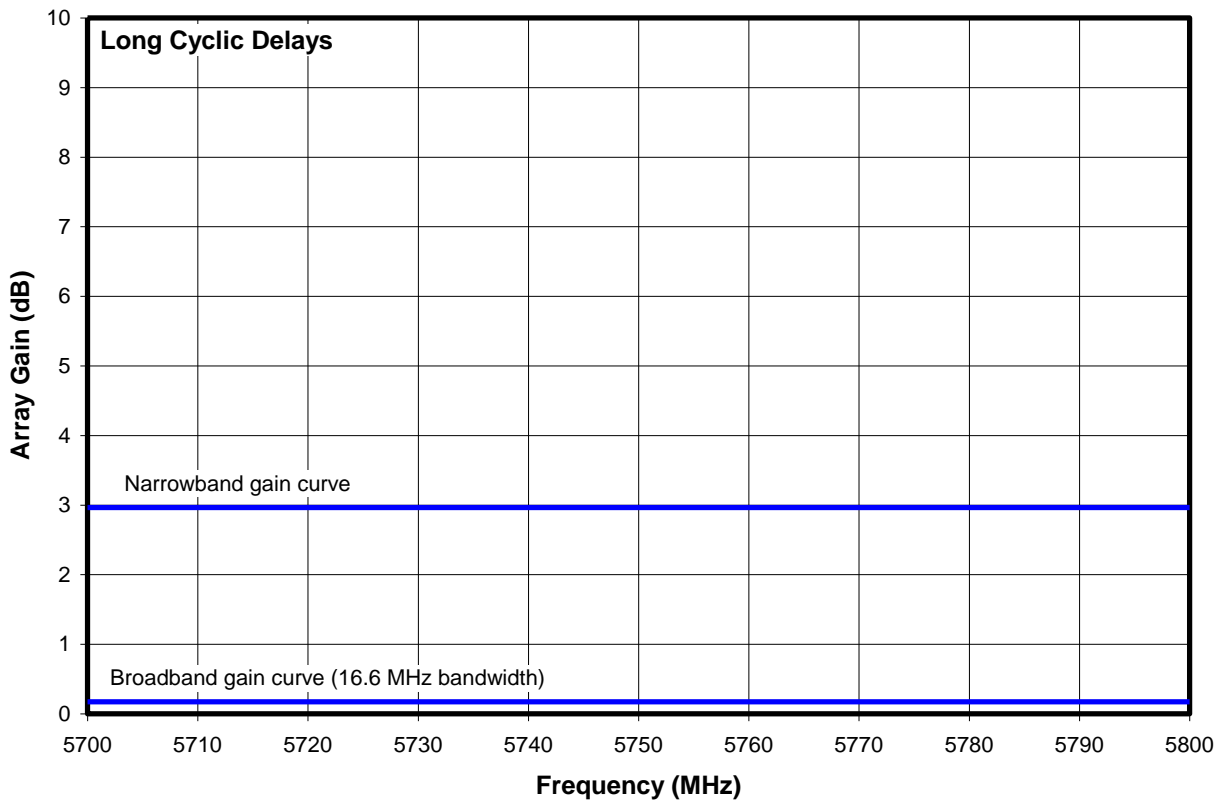
(a) Short Cyclic Delays (Table 1)

(b) Long Cyclic Delays (Table 2)

Figure 5. Broadband 16.6-MHz Azimuth Beam Patterns (5700-5800 MHz) of Four-Antenna Rectangle-2

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(a) Short Cyclic Delays

(b) Long Cyclic Delays

Figure 6. Array Gain Versus Center Frequency (5700-5800 MHz) For an Array With Two Antennas

(a) Short Cyclic Delays

(b) Long Cyclic Delays

Figure 7. Array Gain Versus Center Frequency (5700-5800 MHz) For Arrays With Four Antennas
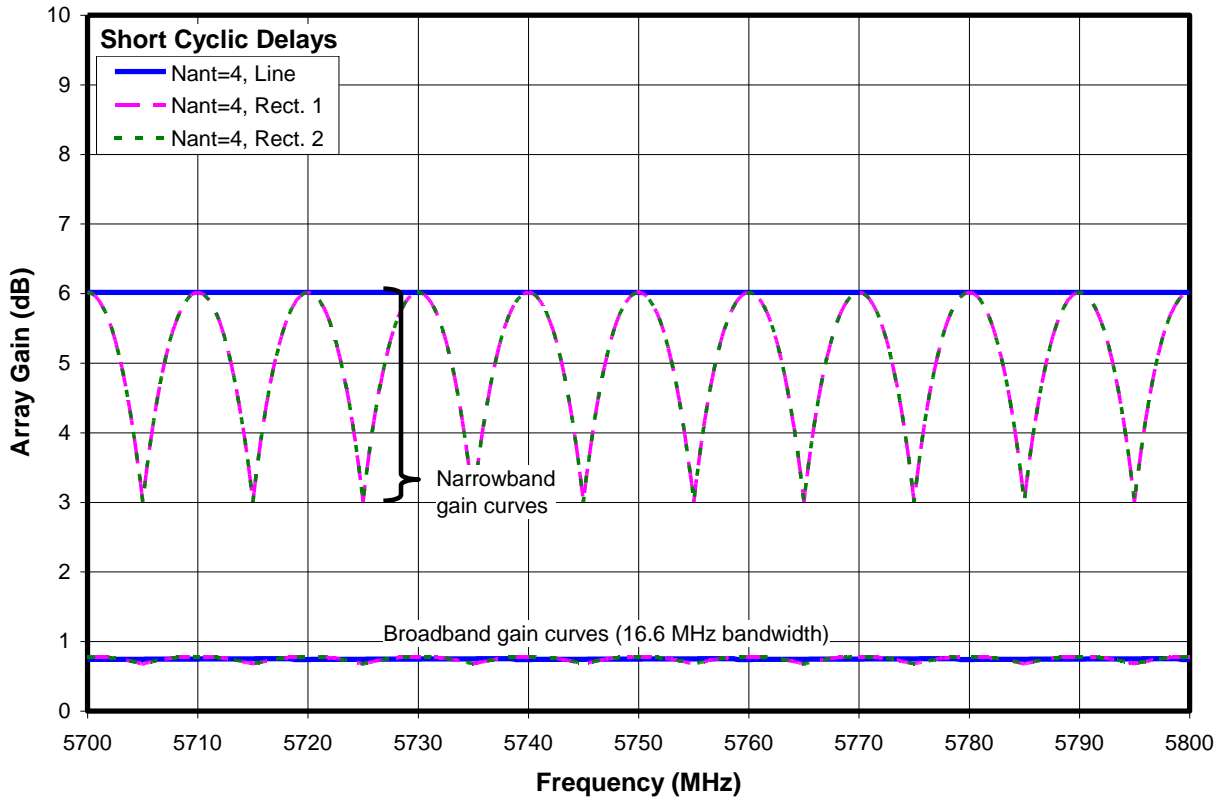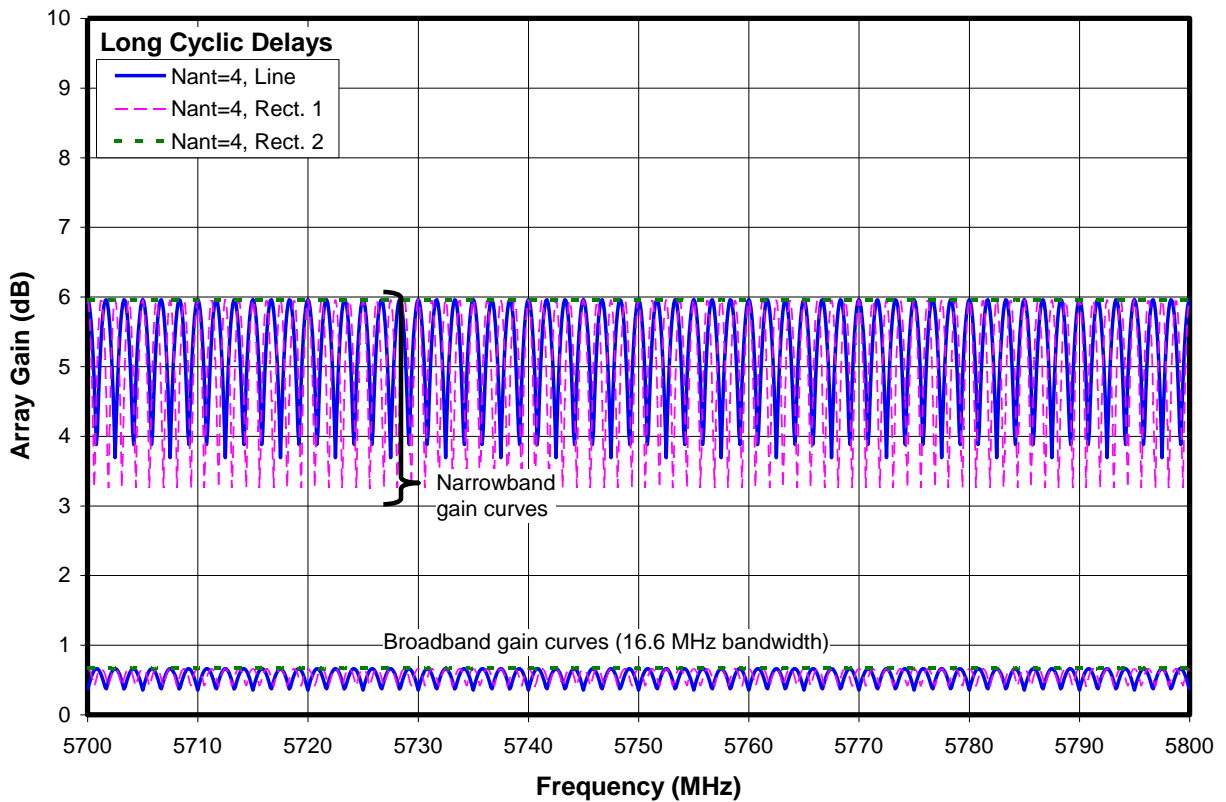
25
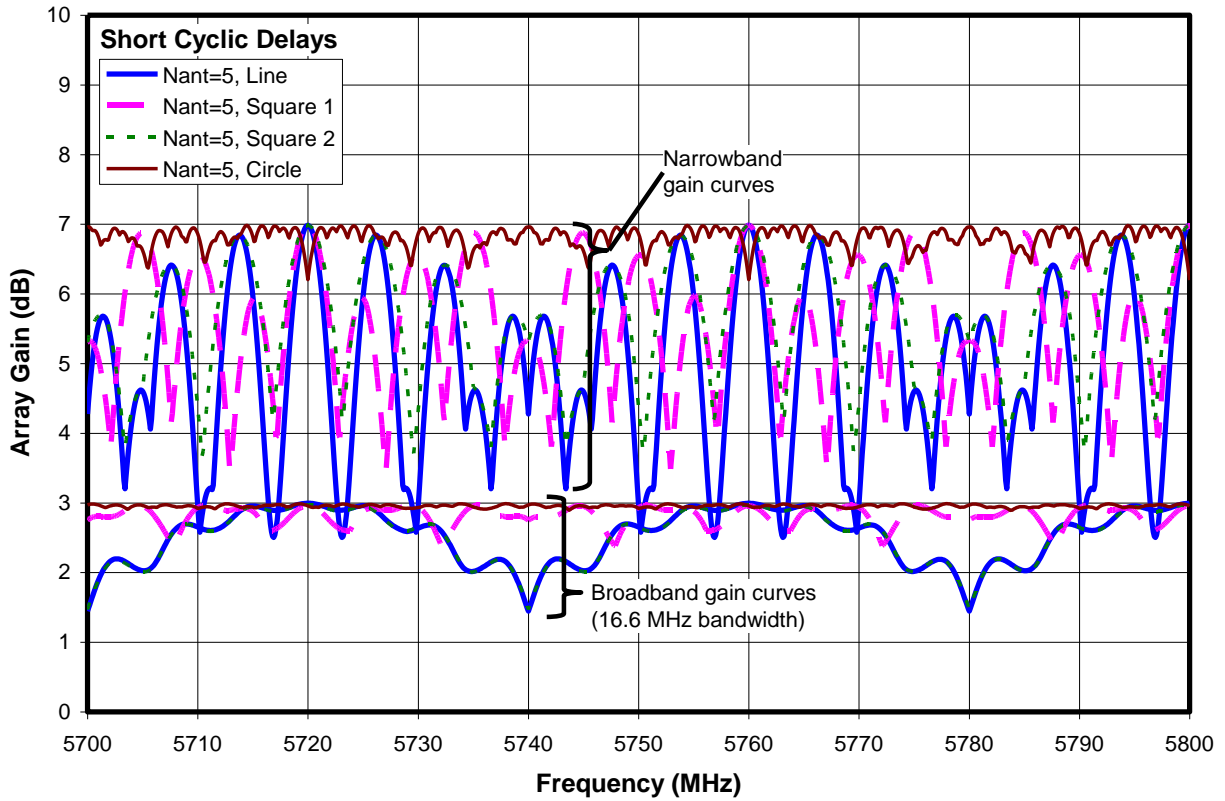
(a) Short Cyclic Delays
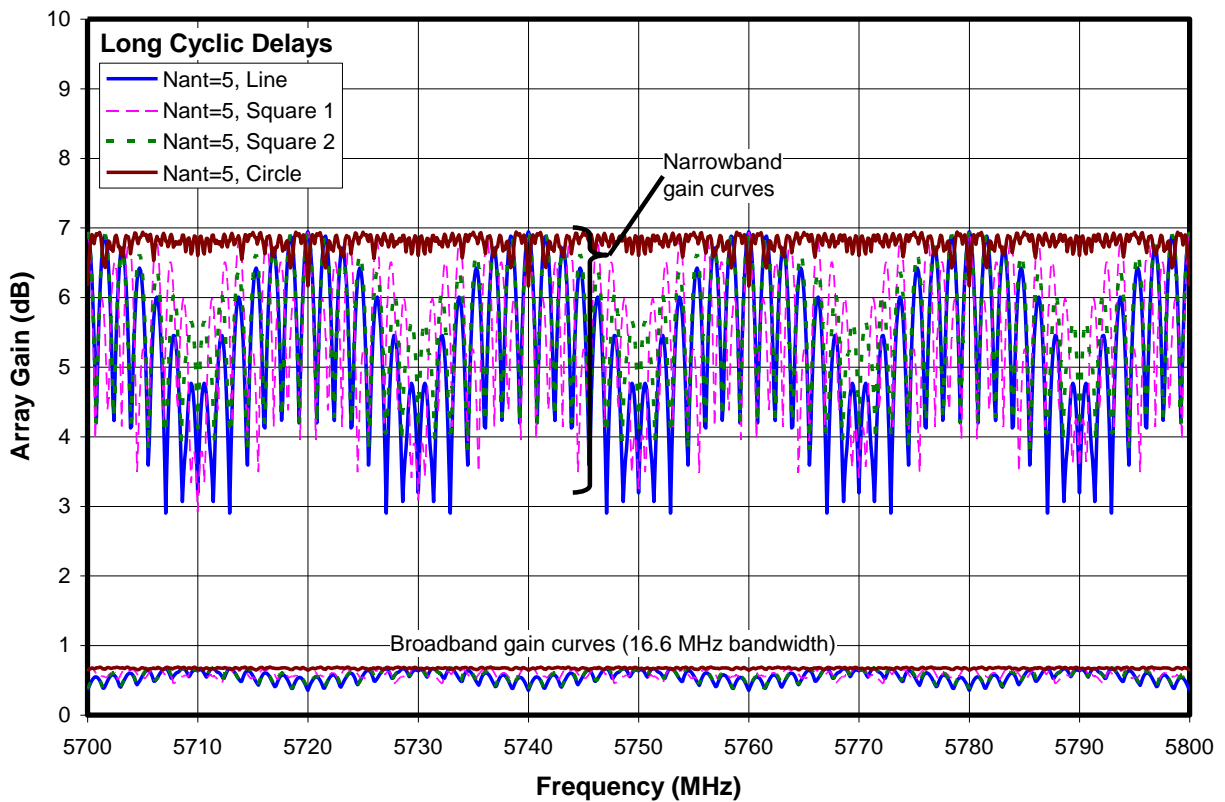
(b) Long Cyclic Delays

Figure 8. Array Gain Versus Center Frequency (5700-5800 MHz) For Arrays with Five Antennas
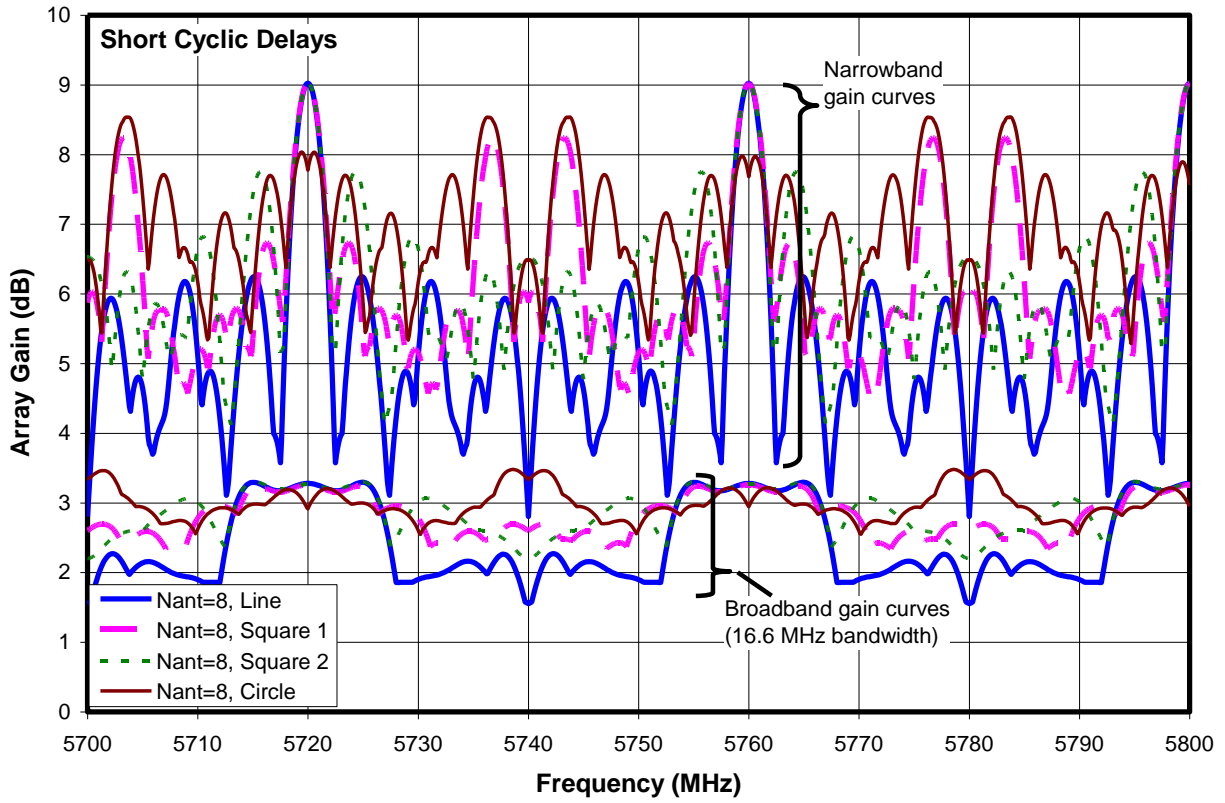
(a) Short Cyclic Delays
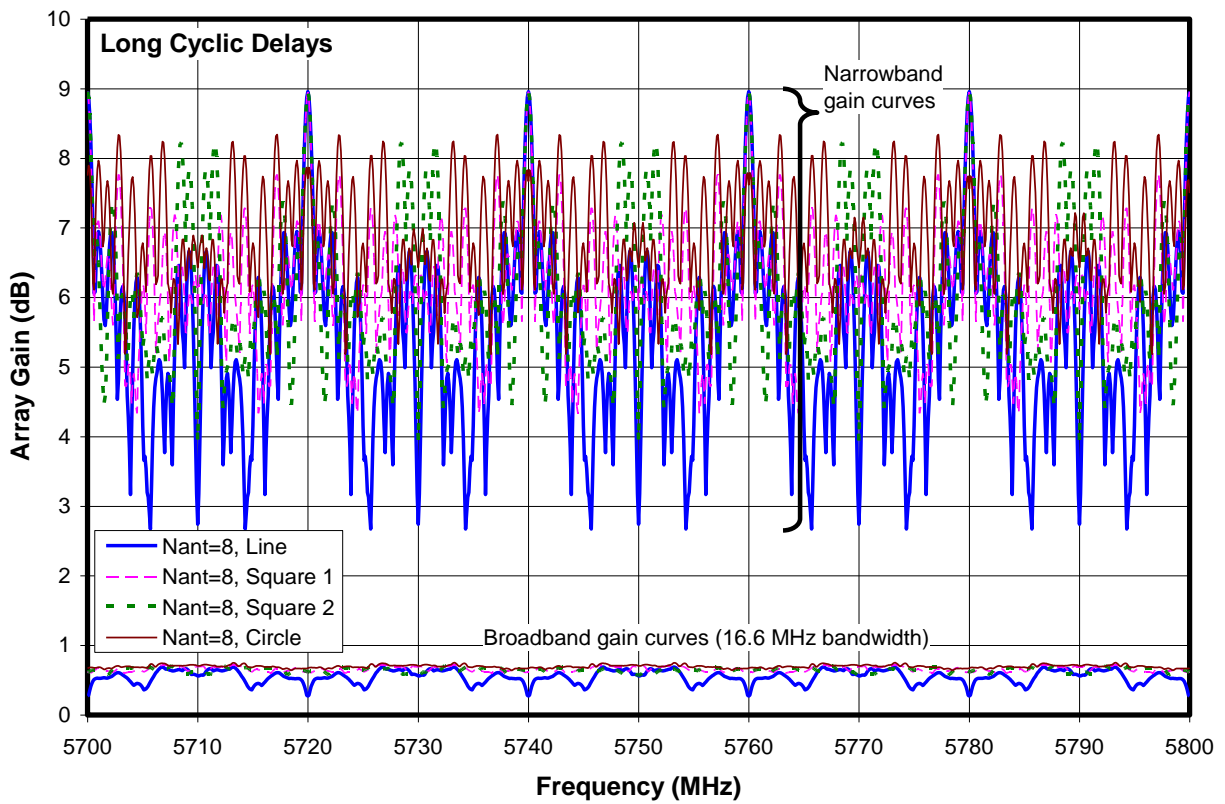
(b) Long Cyclic Delays

Figure 9. Array Gain Versus Center Frequency (5700-5800 MHz) For Arrays With Eight Antennas

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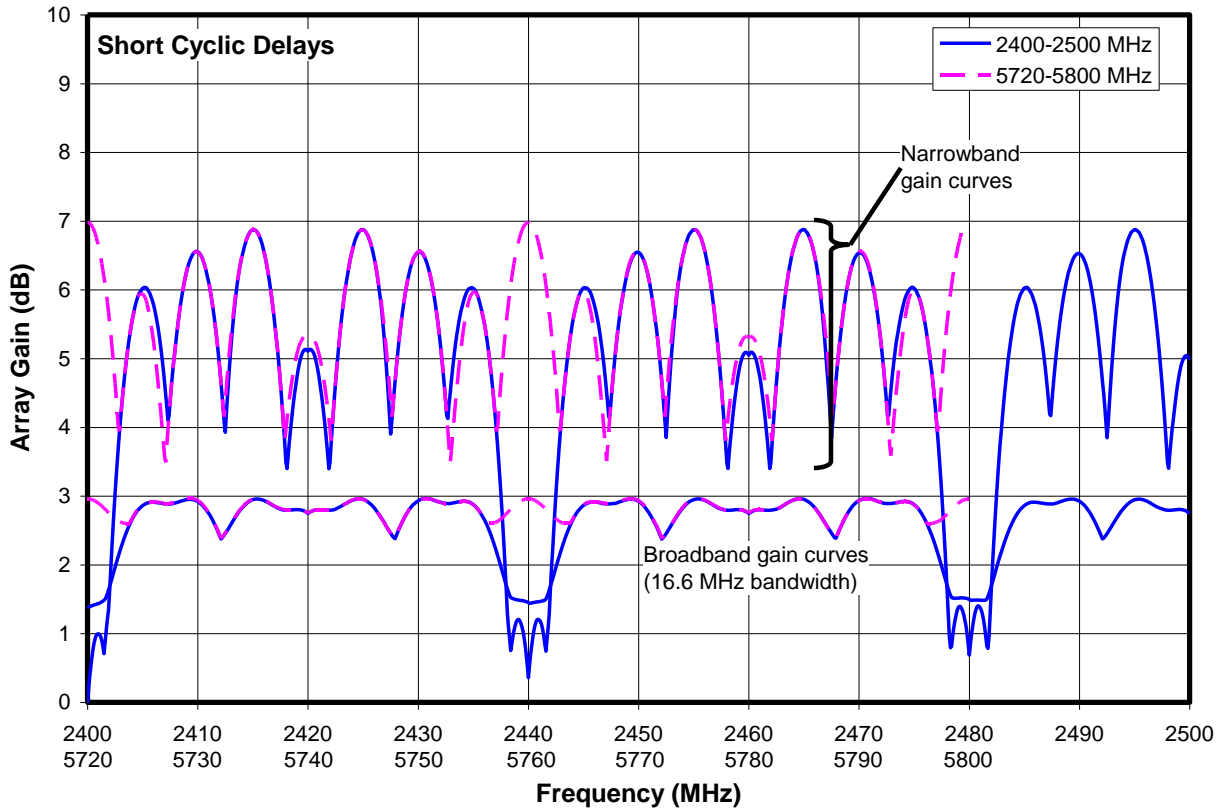
(a) Short Cyclic Delays
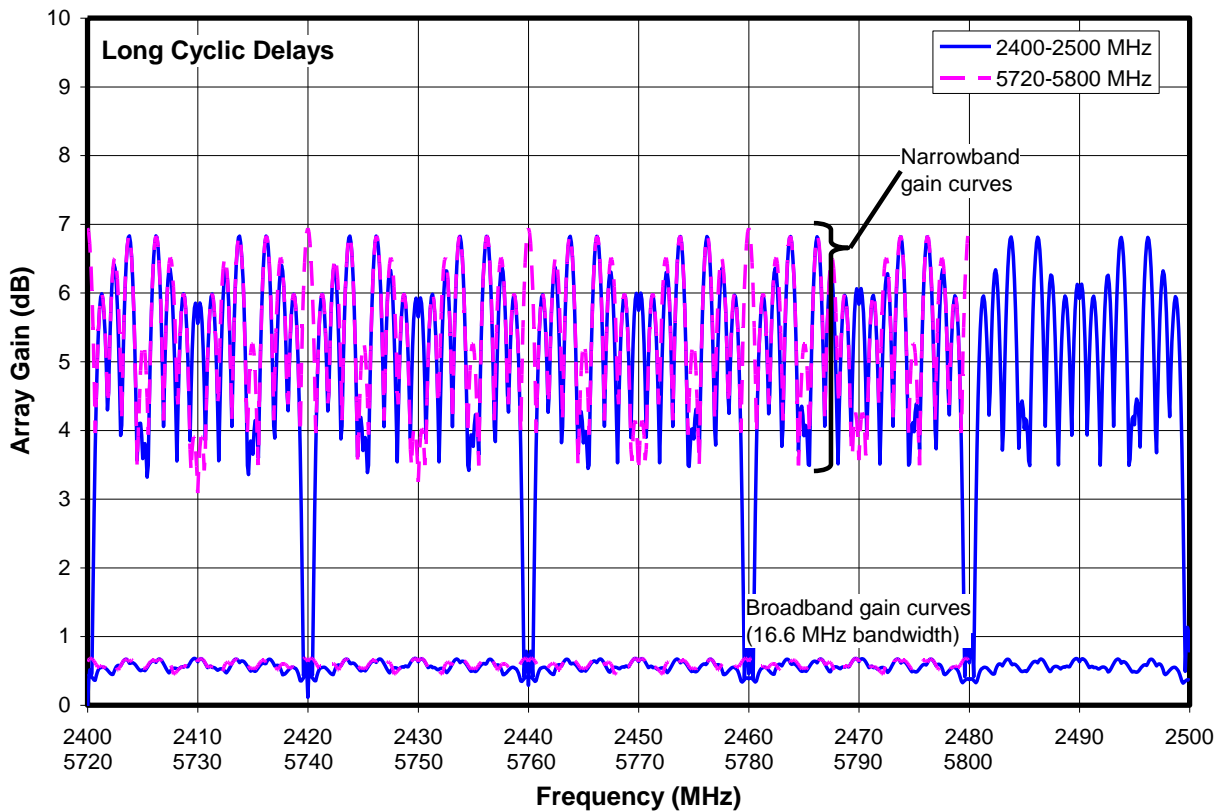
(b) Long Cyclic Delays

Figure 10. Array Gain Versus Center Frequency in Two Frequency Bands For 5-Antenna Square-1

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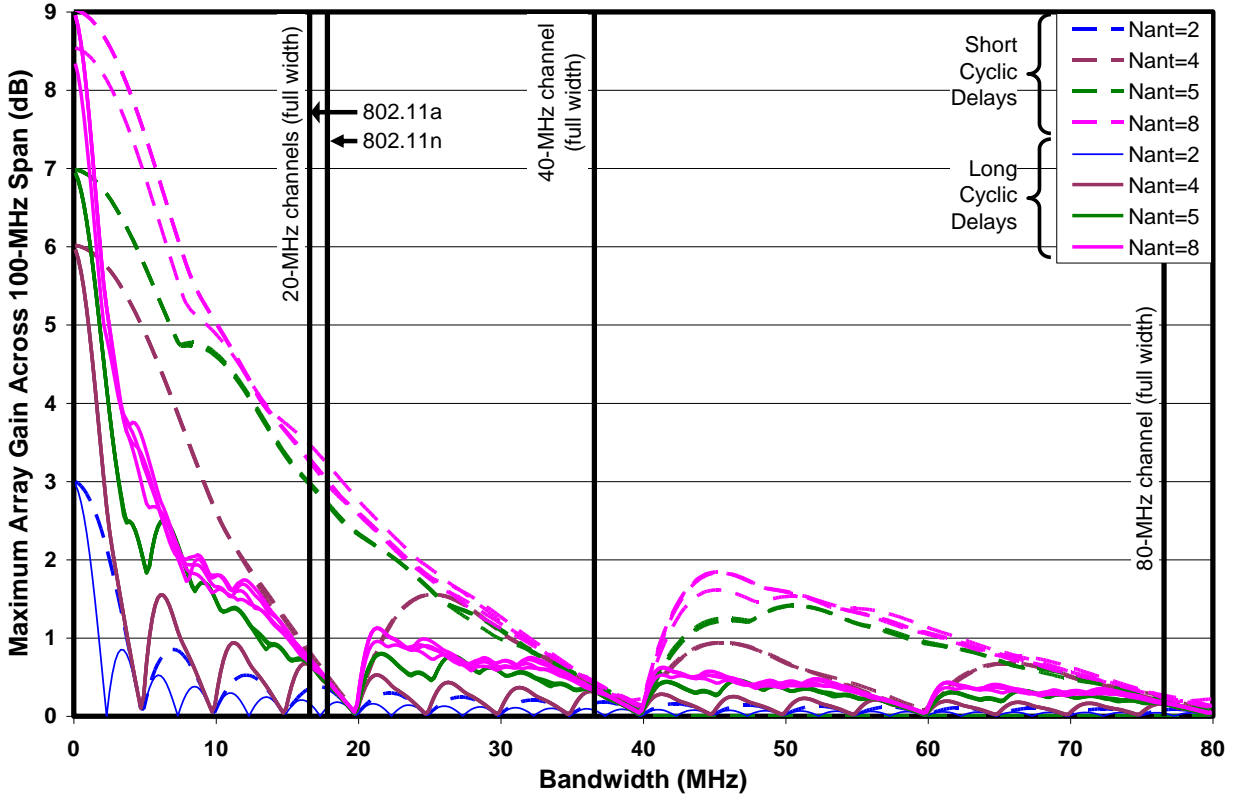
Figure 11. Maximum Array Gain Across 5700-5800 MHz Center Frequency Span Versus Bandwidth
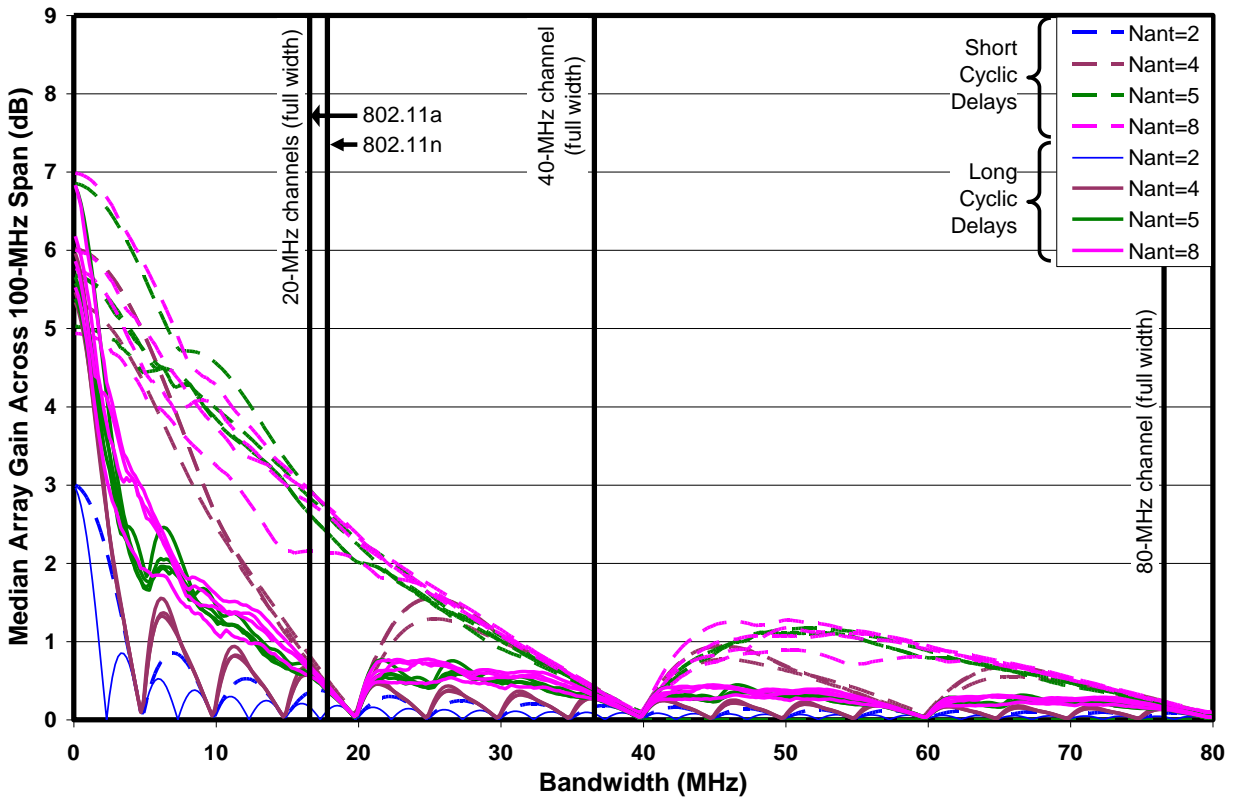
Figure 12. Median Array Gain Across 5700-5800 MHz Center Frequency Span Versus Bandwidth
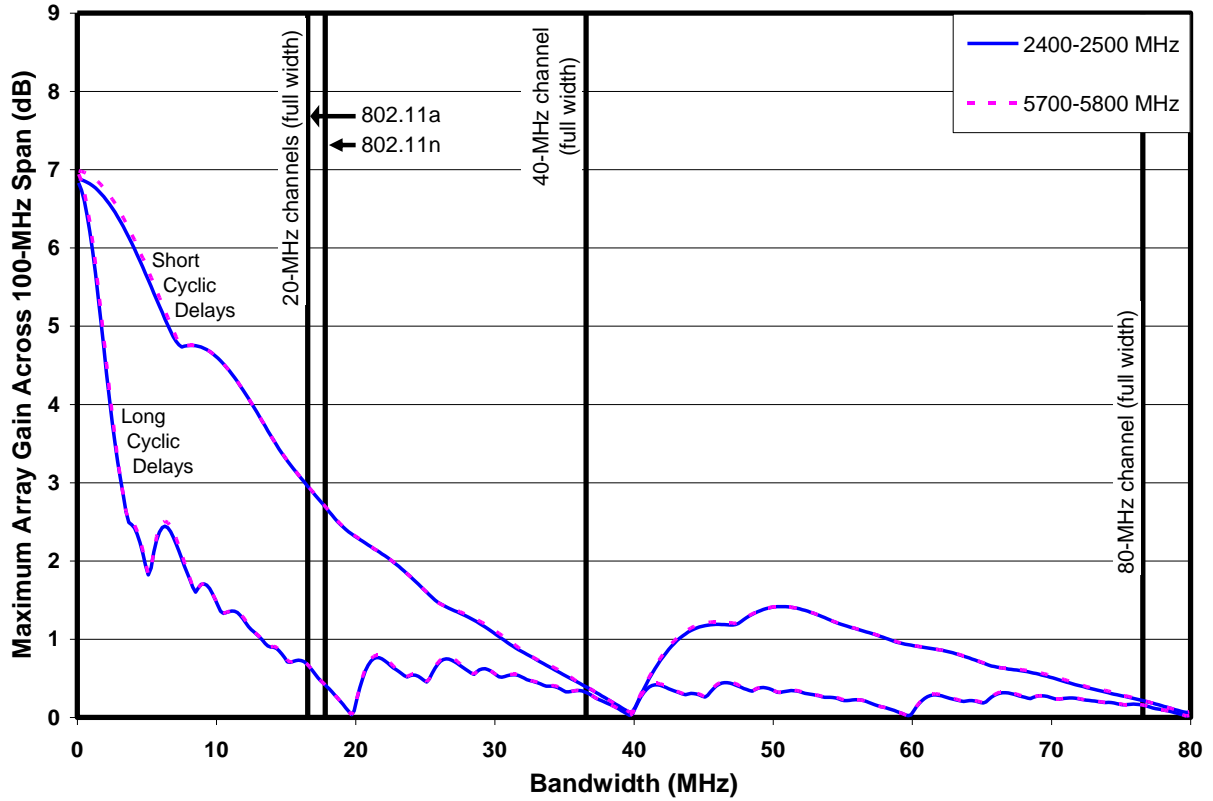
Figure 13. Maximum Array Gain Versus Bandwidth in Two Frequency Bands For 5-Antenna Square-1
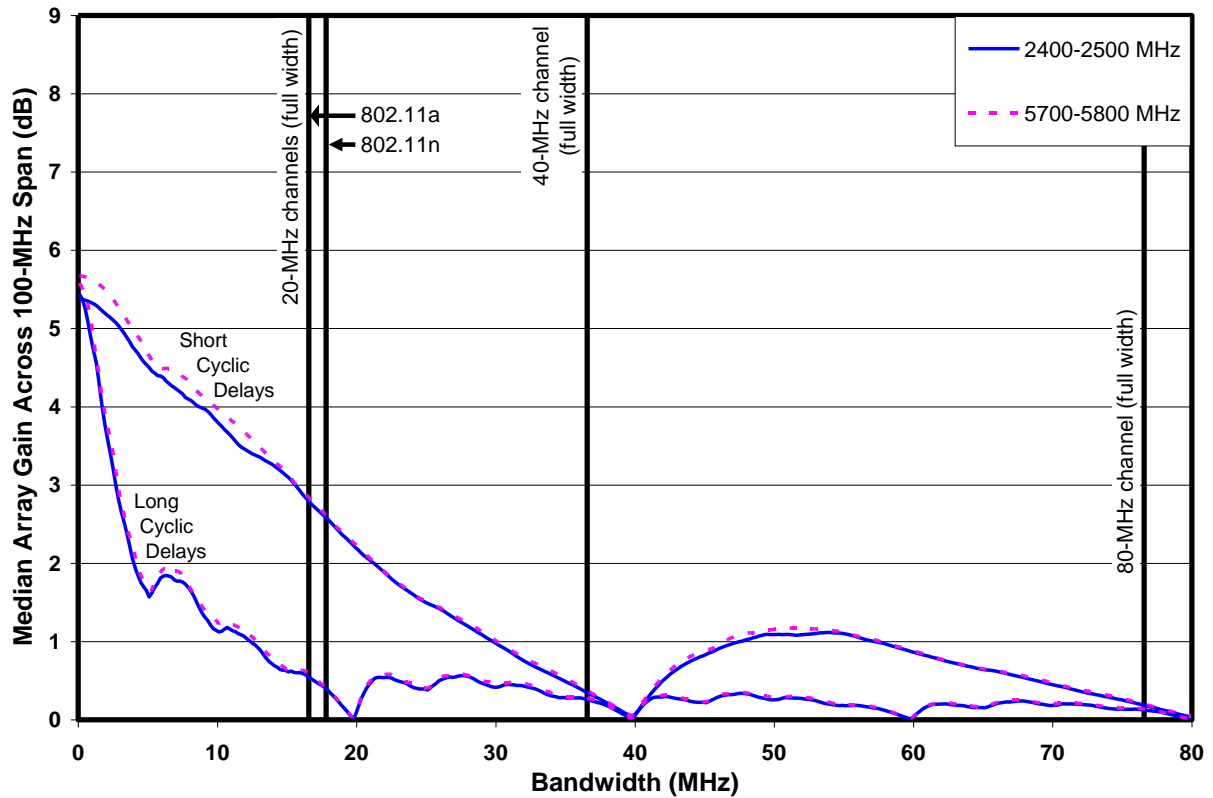
Figure 14. Median Array Gain Versus Bandwidth in Two Frequency Bands For 5-Antenna Square-1
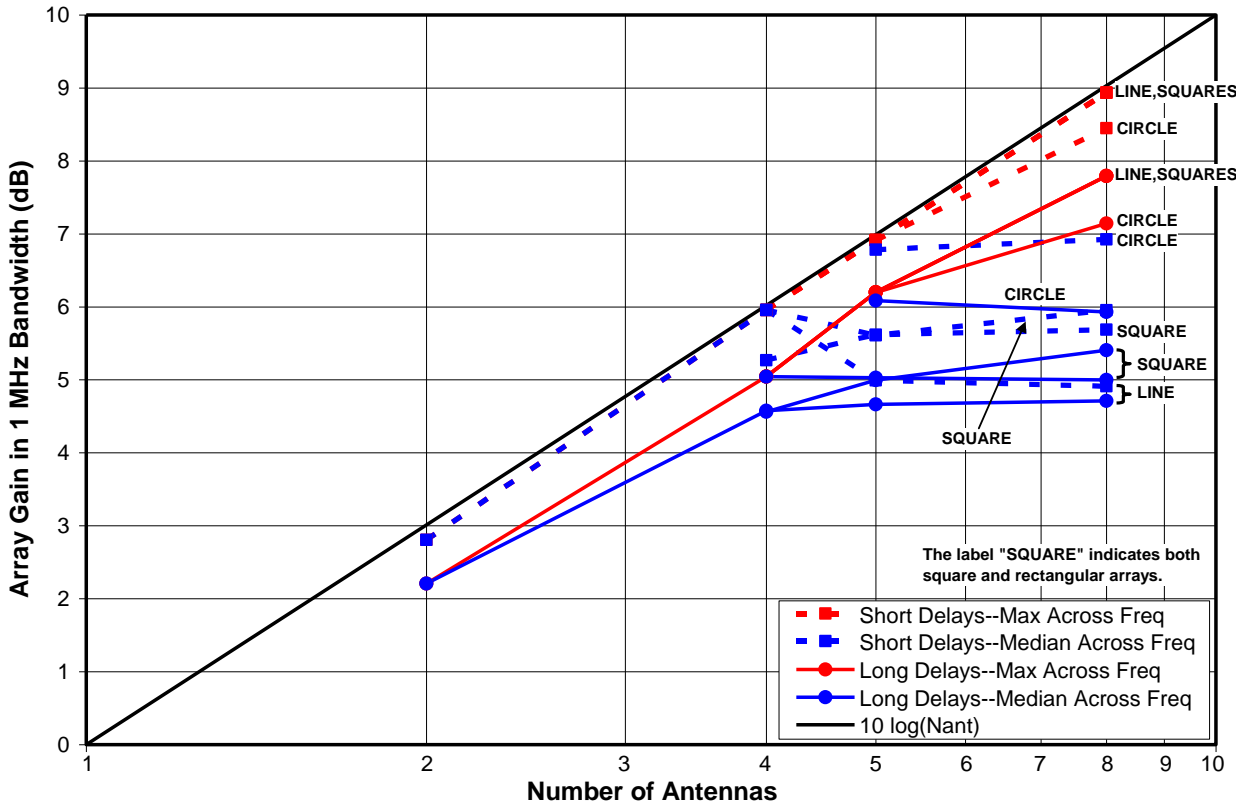
Figure 15. Maximum and Median Array Gains in 1 MHz Bandwidth
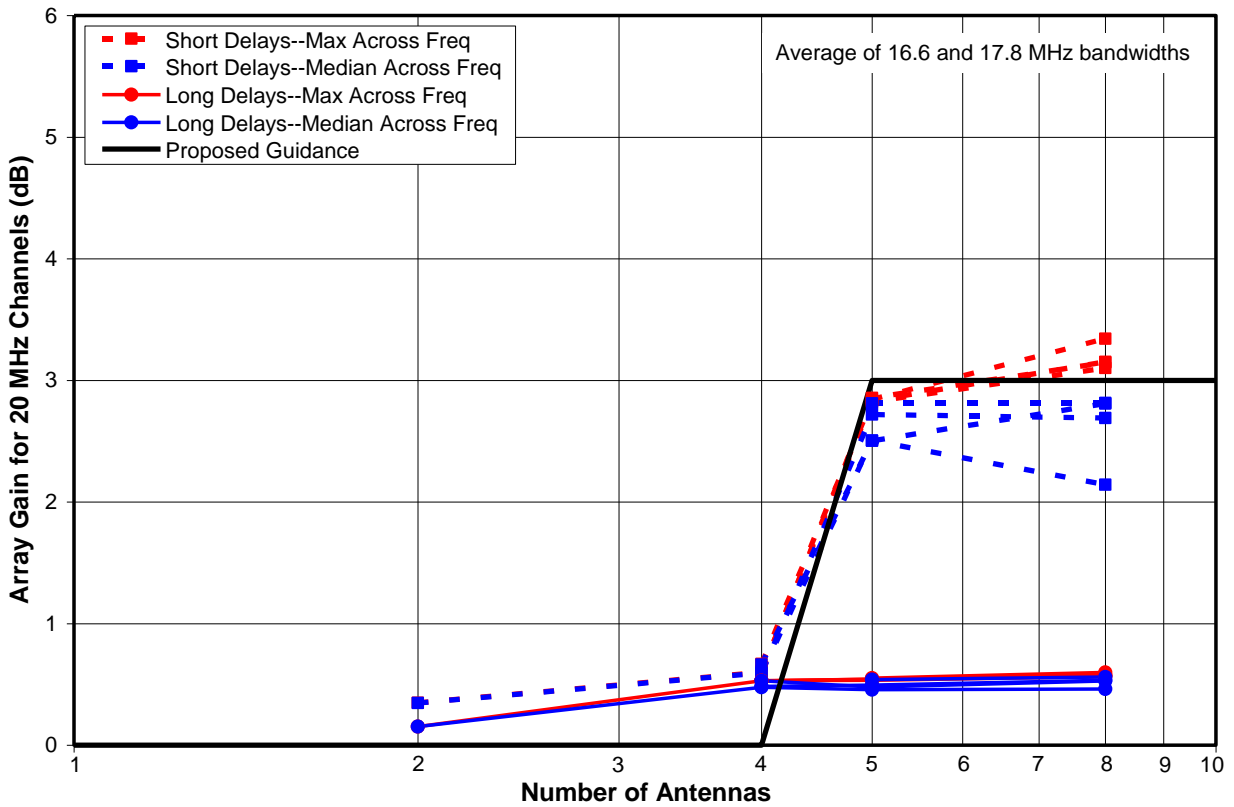
Figure 16. Maximum and Median Broadband Array Gains for 20-MHz Channels

Figure 17. IEEE 802.11 Channels that Straddle FCC Band Boundaries
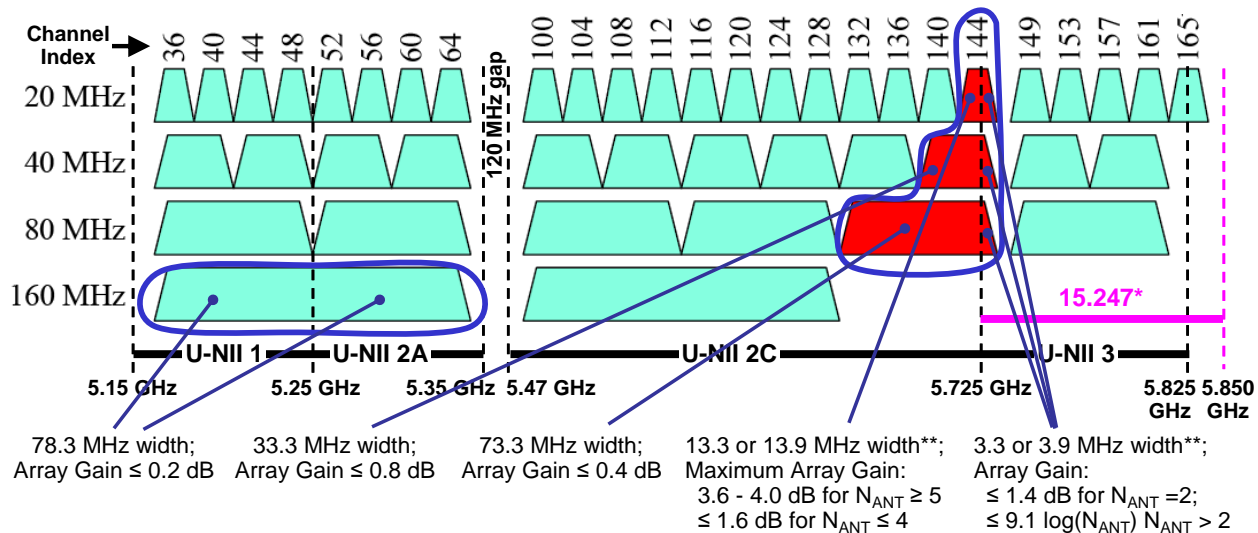
* As of this writing, the FCC's Notice of Proposed Rulemaking FCC13-22 of February 20, 2013 proposes to eliminate digitally modulated operations (while retaining frequency hopping) under 47 CFR 15.247 in the 5.725-5.850 GHz band and to extend the upper edge of the U-NII 3 band under 47 CFR 15.407 to 5.850 GHz.

** 20 MHz 802.11a channels are split into 13.3 MHz and 3.3 MHz portions in U-NII 2C and U-NII 3 bands, respectively. 20 MHz 802.11n or ac channels (HT or VHT) are split into 13.9 and 3.9 MHz portions. The segment of a 40 or 80 MHz channel to the right of the band boundary is 3.3 MHz wide.

Figure 18. Maximum Array Gain Versus Bandwidth With Straddle-Channel Bandwidths Identified

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APPENDIX A
MATLAB CODE

The software shown here was written by the author specifically for this study. The first program computes and plots azimuth beam patterns.

The second program computes frequency-dependent beam patterns over a span of 0 - 360° azimuth and 0 - 45° elevation, smears the beam patterns over bandwidths ranging from 0 – 80 MHz, and identifies the peak response (across azimuth and elevation angle) of the beam pattern to determine array gain. This process is repeated for a range of bandwidths and a range of center frequencies across a selected frequency span. The results are tabulated in a spreadsheet.

In both programs, selection of number of antennas, array configuration, cyclic delays, and frequency band are selected by uncommenting or commenting-out lines of code.

MIMOin802_11vsGainMap.m

```
%CDD MIMO in 802.11--Beam Patterns

clear; clc
%UNITS
    %Dimensions in m
    %Frequency in MHz
    %Time delays in microseconds
    %c in m/us
%CONSTANTS
    c=300 % m/us
%ARRAY DESCRIPTION
    N=8
    % # of antennas
    if N==2
        a=0.1
        x=[0;a]; y=[0;0]
    end
    if N==4
        %Line Array
            a=0.1 % meters
            x=[0;a;2*a;3*a]; y=[0;0;0;0]
        %Rectangular Array
            %a=0.175, b=0.109 % in m
            %x=[0;a;a;0]; y=[0;0;b;b]
        %Rectangular Array 2
            %a=0.175, b=0.109 % in m
            %x=[0;a;0;a]; y=[0;0;b;b]
        %Other Versions of Rectangular Array:
            %x=[0;a;0;a]; y=[0;0;b;b]
            %x=[0;a;a;0]; y=[0;0;b;b]
            %x=[0;a;0;a]; y=[0;b;b;0]
            %x=[0;b;b;0]; y=[0;0;a;a]
            %x=[0;b;0;b]; y=[0;0;a;a]
            %x=[0;a;a;0]; y=[0;b;0;b]
            %x=[0;0.06;0.12;0.18]; y=[0;0;0;0]
```

```
      end
   if N==5
      %Line Array
         %a=0.06 % meters
         %x=[0;a;2*a;3*a;4*a]; y=[0;0;0;0;0]
      %Square array
         %a=0.1 %meters
         %x=[0;a;a;0;0.5*a]; y=[0;0;a;a;0.5*a]
      %Square array2
         a=0.1 %meters
         x=[0;a;0.5*a;0;a]; y=[0;0;0.5*a;a;a]
      %Circular array
         %r=0.12 %meters radius;
         %a=2*pi/N
         %x=[r*cos(0);r*cos(a);r*cos(2*a);r*cos(3*a);r*cos(4*a)];
         %y=[r*sin(0);r*sin(a);r*sin(2*a);r*sin(3*a);r*sin(4*a)];
   end
   if N==6
      %Line Array
         %a=0.06 % meters
         %x=[0;a;2*a;3*a;4*a;5*a]; y=[0;0;0;0;0;0]
      %Circular array
         r=0.12 %meters radius;
         a=2*pi/N


x=[r*cos(0);r*cos(a);r*cos(2*a);r*cos(3*a);r*cos(4*a);r*cos(5*a)];


y=[r*sin(0);r*sin(a);r*sin(2*a);r*sin(3*a);r*sin(4*a);r*sin(5*a)];
   end
   if N==8
      %Line Array
         %a=0.06 % meters
         %x=[0;a;2*a;3*a;4*a;5*a;6*a;7*a]; y=[0;0;0;0;0;0;0;0]
      %Square array
         a=0.1 %meters
         x=[0;a;2*a;2*a;2*a;a;0;0]; y=[0;0;0;a;2*a;2*a;2*a;a]
      %Square array2
         %a=0.1 %meters
         %x=[0;a;2*a;0;2*a;0;a;2*a]; y=[0;0;0;a;a;2*a;2*a;2*a]
      %Circular array
         %r=0.12 %meters radius;
         %a=2*pi/N


%x=[r*cos(0);r*cos(a);r*cos(2*a);r*cos(3*a);r*cos(4*a);r*cos(5*a);r*cos(6*a)

;r*cos(7*a)];


%y=[r*sin(0);r*sin(a);r*sin(2*a);r*sin(3*a);r*sin(4*a);r*sin(5*a);r*sin(6*a)

;r*sin(7*a)];
   end
   %Compute array element locations in units of propagation delay at light

speed
      xt=x/c; yt=y/c;
```

```
    %Compute maximum array dimensions to be used in setting azimuth angle
    %increment for analysis:
        ArrayAperture=sqrt((max(x)-min(x))^2+(max(y)-min(y))^2)
        ArrayTransitTime=ArrayAperture/c
%MIMO DESCRIPTION (time delays in microseconds)
    MimoDelayGroup=1
    if MimoDelayGroup==1
        if N==2; Tmimo=[0;0.200];end
        if N==3; Tmimo=[0;0.100;0.200];end
        if N==4; Tmimo=[0;0.050;0.100;0.150];end
        if N==5; Tmimo=[0;0.175;0.025;0.050;0.075];end
        if N==6; Tmimo=[0;0.200;0.025;0.150;0.175;0.125];end
        if N==7; Tmimo=[0;0.200;0.150;0.025;0.175;0.075;0.050];end
        if N==8; Tmimo=[0;0.175;0.150;0.125;0.025;0.100;0.050;0.200];end
    end
    if MimoDelayGroup==2
        if N==2; Tmimo=[0;0.400];end
        if N==3; Tmimo=[0;0.400;0.200];end
        if N==4; Tmimo=[0;0.400;0.200;0.600];end
        if N==5; Tmimo=[0;0.400;0.200;0.600;0.350];end
        if N==6; Tmimo=[0;0.400;0.200;0.600;0.350;0.650];end
        if N==7; Tmimo=[0;0.400;0.200;0.600;0.350;0.650;0.100];end
        if N==8; Tmimo=[0;0.400;0.200;0.600;0.350;0.650;0.100;0.750];end
    end
%FREQUENCIES AND BANDS
    %Bands for analysis:
        %(Consecutive FCC bands are combined). Band 4 is for debugging
        Bands=[2402,2482; ... %1
            5170,5250; ... %2 Channel range in U-NII 1
            5250,5330; ... %3 Channel range in U-NII 2
            5490,5730; ... %4 Channel range in U-NII 2ext+channel 144
            5735,5835; ... %5 Channel range in 15.247 5.8 GHz band
            5170,5330; ... %6 Channel range of combined lower UNII bands
            5150,5350; ... %7 Combined lower UNII bands
            5470,5850; ... %8 Combined upper UNII bands
            5730,5780; ... %9 40 MHz channel for beam pattern test on

sample A
            5700,5800; ... %10 100 MHz wide band for analysis
            5150,5170; ... %11 Narrower band for S/W diagnostics
            5500,5800];   %12 Narrower band and low freq (larger angle

increment) for S/W diagnostics

        SelectedBand=10 %******************************
        Band=Bands(SelectedBand,:)'
    CenterFreq=Band(1):0.1:Band(2);
%Bandwidth for Array Gain Calculation
    Bandwidth = 0.1 %MHz (narrowband)
    %Bandwidth=1 %MHz (PSD)
    %Bandwidth=16.25 %MHz (20 MHz channel)
    %Bandwidth=44*0.3125 %MHz (Part of 20 MHz HT chan 144 in UNII-2ext)
    %Bandwidth=3.75 %MHz (Part of 20 MHz HT chan 144 in UNII-3)
    %Bandwidth=36.25 %MHz (40 MHz channel), but channel center freq will

differ
%Increment Sizes
    Fmax=max(Band) %MHz
    LambdaMin=c/Fmax;
```

```
    NumPhaseStepsIn360=360/15;
    %Azimuth Increment
      ApproxAngleIncrRadians=(LambdaMin/NumPhaseStepsIn360)/ArrayAperture;
      NAngleSteps=1+round(2*pi/ApproxAngleIncrRadians);
      AngleIncrRadians=2*pi/NAngleSteps;
      AzimuthRadians=AngleIncrRadians*(0:NAngleSteps);
%Array Gain Calculations
    BBGainAtAz=zeros(NAngleSteps+1,1);
    BBGain=zeros(length(BBGainAtAz),length(CenterFreq));
    FIncr=0.1 %MHz
    Halfspan=(Bandwidth-FIncr)/2
    for Ichan=1:length(CenterFreq)
      F=CenterFreq(Ichan)-Halfspan:FIncr:CenterFreq(Ichan)+Halfspan;
      for IAz=1:NAngleSteps+1


Gain=abs(sum(exp(1i*2*pi*(Tmimo+cos(AzimuthRadians(IAz))*xt+sin(AzimuthRadia

ns(IAz))*yt)*F))).^2/N;
        BBGainAtAz(IAz,Ichan)=10*log10(mean(Gain)); %Average the gain

across bandwidth
      end
    end
%PLOT:  Color map w/contours
    figure(1);clf
    %surf(ChanFreq,AzimuthRadians*180/pi,BBGainAtAz)
    colormap jet
    %colormap hot
    GainScale=round(10*log10(N))
    ContourLevels=[0:1:GainScale]
    pcolor(CenterFreq,AzimuthRadians*180/pi,BBGainAtAz)
    caxis([-3 GainScale])
    shading flat
    hold on
    contour(CenterFreq,AzimuthRadians*180/pi,BBGainAtAz,ContourLevels,'k');
    hold on
    contour(CenterFreq,AzimuthRadians*180/pi,BBGainAtAz,[0

10],'k','LineWidth',2);
    %Tick marks and Colorbar legend
      set(gca,'XTick',[Band(1):20:Band(2)])
      xlim(Band)
      set(gca,'XTick',[Band(1):20:Band(2)])
      set(gca,'TickDir','out')
      set(gca,'YTick',[0:90:360])
      xlabel('Center Frequency (MHz)','Fontsize',13)
      ylabel('Azimuth (degrees)','Fontsize',13)
      temp=colorbar;  %Insert colorbar and get its handle
      set(get(temp,'ylabel'),'string','Array Gain (dB)','Fontsize',13)
%PLOT: Max Gain Across Az vs Frequency
    figure(2);clf
    MaxBBGainAcrossAz=max(BBGainAtAz);
    plot(CenterFreq,MaxBBGainAcrossAz)
    %Plot scaling, tick marks and labels
      xlim(Band)
      ylim([0 GainScale+1])
      set(gca,'XTick',[Band(1):20:Band(2)])
      set(gca,'YTick',[0:1:GainScale])
```

```
  grid on
  xlabel('Center Frequency (MHz)','Fontsize',13)
  ylabel('Array Gain (dBi)','Fontsize',13)
```

MIMOin802_11vsFreqAndMultipleBW_q.m

```
%CDD MIMO in 802.11--Broadband Array Gain

clear; clc
%UNITS
   %Dimensions in m
   %Frequency in MHz
   %Time delays in microseconds
   %c in m/us
%CONSTANTS
   c=300 % m/us
%ARRAY DESCRIPTION
   N=5 % # of antennas
   if N==2
      a=0.1
      x=[0;a]; y=[0;0]
   end
   if N==4
      %Line Array
         %a=0.1 % meters
         %x=[0;a;2*a;3*a]; y=[0;0;0;0]
      %Rectangular Array
         %a=0.175, b=0.109 % in m
         %x=[0;a;a;0]; y=[0;0;b;b]
      %Rectangular Array 2
         a=0.175, b=0.109 % in m
         x=[0;a;0;a]; y=[0;0;b;b]
      %Other Versions of Rectangular Array:
         %x=[0;a;0;a]; y=[0;0;b;b]
         %x=[0;a;a;0]; y=[0;0;b;b]
         %x=[0;a;0;a]; y=[0;b;b;0]
         %x=[0;b;b;0]; y=[0;0;a;a]
         %x=[0;b;0;b]; y=[0;0;a;a]
         %x=[0;a;a;0]; y=[0;b;0;b]
         %x=[0;0.06;0.12;0.18]; y=[0;0;0;0]
   end
   if N==5
      %Line Array
         %a=0.06 % meters
         %x=[0;a;2*a;3*a;4*a]; y=[0;0;0;0;0]
      %Square array
         %a=0.1 %meters
         %x=[0;a;a;0;0.5*a]; y=[0;0;a;a;0.5*a]
      %Square array2
         %a=0.1 %meters
         %x=[0;a;0.5*a;0;a]; y=[0;0;0.5*a;a;a]
      %Circular array
         r=0.12 %meters radius;
         a=2*pi/N
         x=[r*cos(0);r*cos(a);r*cos(2*a);r*cos(3*a);r*cos(4*a)];
         y=[r*sin(0);r*sin(a);r*sin(2*a);r*sin(3*a);r*sin(4*a)];
   end
```

```matlab
if N==6
  %Line Array
     %a=0.06 % meters
     %x=[0;a;2*a;3*a;4*a;5*a]; y=[0;0;0;0;0;0]
  %Circular array
     r=0.12 %meters radius;
     a=2*pi/N
     x=[r*cos(0);r*cos(a);r*cos(2*a);r*cos(3*a);r*cos(4*a);r*cos(5*a)];
     y=[r*sin(0);r*sin(a);r*sin(2*a);r*sin(3*a);r*sin(4*a);r*sin(5*a)];
end
if N==8
  %Line Array
     %a=0.06 % meters
     %x=[0;a;2*a;3*a;4*a;5*a;6*a;7*a]; y=[0;0;0;0;0;0;0;0]
  %Square array
     %a=0.1 %meters
     %x=[0;a;2*a;2*a;2*a;a;0;0]; y=[0;0;0;a;2*a;2*a;2*a;a]
  %Square array2
     a=0.1 %meters
     x=[0;a;2*a;0;2*a;0;a;2*a]; y=[0;0;0;a;a;2*a;2*a;2*a]
  %Circular array
     %r=0.12 %meters radius;
     %a=2*pi/N
     %x=[r*cos(0);r*cos(a);r*cos(2*a);r*cos(3*a);r*cos(4*a);r*cos(5*a);r*cos(6*a);r*cos(7*a)];
     %y=[r*sin(0);r*sin(a);r*sin(2*a);r*sin(3*a);r*sin(4*a);r*sin(5*a);r*sin(6*a);r*sin(7*a)];
end
%Compute array element locations in units of propagation delay at light speed
  xt=x/c; yt=y/c;
%Compute maximum array dimensions to be used in setting azimuth angle
%increment for analysis:
  ArrayAperture=sqrt((max(x)-min(x))^2+(max(y)-min(y))^2)
  ArrayTransitTime=ArrayAperture/c
%MIMO DESCRIPTION (time delays in microseconds)
  MimoDelayGroup=1
  if MimoDelayGroup==1
    if N==2; Tmimo=[0;0.200];end
    if N==3; Tmimo=[0;0.100;0.200];end
    if N==4; Tmimo=[0;0.050;0.100;0.150];end
    if N==5; Tmimo=[0;0.175;0.025;0.050;0.075];end
    if N==6; Tmimo=[0;0.200;0.025;0.150;0.175;0.125];end
    if N==7; Tmimo=[0;0.200;0.150;0.025;0.175;0.075;0.050];end
    if N==8; Tmimo=[0;0.175;0.150;0.125;0.025;0.100;0.050;0.200];end
  end
  if MimoDelayGroup==2
    if N==2; Tmimo=[0;0.400];end
    if N==3; Tmimo=[0;0.400;0.200];end
    if N==4; Tmimo=[0;0.400;0.200;0.600];end
    if N==5; Tmimo=[0;0.400;0.200;0.600;0.350];end
    if N==6; Tmimo=[0;0.400;0.200;0.600;0.350;0.650];end
    if N==7; Tmimo=[0;0.400;0.200;0.600;0.350;0.650;0.100];end
    if N==8; Tmimo=[0;0.400;0.200;0.600;0.350;0.650;0.100;0.750];end
  end
%FREQUENCIES AND BANDS
  %Bands for analysis:
    %(Consecutive FCC bands are combined). Band 4 is for debugging
    Bands=[2402,2482; ... %1
        5170,5250; ... %2 Channel range in U-NII 1
        5250,5330; ... %3 Channel range in U-NII 2
        5490,5730; ... %4 Channel range in U-NII 2ext+channel 144
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            5735,5835; ... %5 Channel range in 15.247 5.8 GHz band
            5170,5330; ... %6 Channel range of combined lower UNII bands
            5150,5350; ... %7 Combined lower UNII bands
            5470,5850; ... %8 Combined upper UNII bands
            5730,5780; ... %9 40 MHz channel for beam pattern test on sample A
            5700,5800; ... %10 100 MHz wide band for analysis
            5150,5170; ... %11 Narrower band for S/W diagnostics
            5500,5800];    %12 Narrower band and low freq (larger angle increment) for S/W diagnostics


        SelectedBand=10 %******************************
        Band=Bands(SelectedBand,:)'
%Bandwidth for Array Gain Calculation
    MaxBandwidth = 80 % MHz ***************************
%Increment Sizes
    Fmax=max(Band) %MHz
    LambdaMin=c/Fmax;
    NumPhaseStepsIn360=360/15;
    %Azimuth Increment
        ApproxAngleIncrRadians=(LambdaMin/NumPhaseStepsIn360)/ArrayAperture;
        NAngleSteps=1+round(2*pi/ApproxAngleIncrRadians);
        AngleIncrRadians=2*pi/NAngleSteps;
        AzimuthRadians=AngleIncrRadians*(0:NAngleSteps-1);
    %Increment of cos(elevation angle):
        %Let elevation angle go 0 to 45 degrees
        MaxElev=45 %degrees
        NCosElevPoints = 1+round((cos(0)-cos(MaxElev*pi/180))/ApproxAngleIncrRadians)
        CosElev=linspace(cos(0),cos(MaxElev*pi/180),NCosElevPoints);
    %Frequency increment:
        FIncr=0.1 %MHz ******************
%Narrowband Array Gain Calculations (Calculate gain vs frequency and azimuth)
    %Number of frequency points in band
        NCenterFreq=1+round((Band(2)-Band(1))/FIncr)
    %Frequency span for narrowband gain analysis will be extended beyond the band to allow
    %subsequent broadband analysis for bands centered at any frequency from band edge to band edge.
    %# of points added to frequency width on each side = Ifreqext
        Ifreqext = round(0.5*MaxBandwidth/FIncr);
        Frequencies = Band(1)- Ifreqext*FIncr + FIncr*(0:NCenterFreq+2*Ifreqext-1);
    %Compute narrowband array gain vs azimuth, elevation, and frequency over the extended frequency range:
        %NBGainAtAz=[Azimuth,Frequency,Cos(Elev)]
        NBGainAtAz=zeros(NAngleSteps,length(Frequencies),length(CosElev));
        for ICosElev=1:length(CosElev);
            ['ICosElev = ' int2str(ICosElev)]
            for IAz=1:NAngleSteps
                %In the expression below, the quanity inside the sum is a
                %complex array of whose elements are arranged as Antenna
                %Element index X Frequency.  The sum function sums across
                %antenna elements:
                %NBGainAtAz=[Azimuth,Frequency,Cos(Elev)]
                NBGainAtAz(IAz,:,ICosElev)=abs(sum(exp(1i*2*pi*...
                    (Tmimo+cos(AzimuthRadians(IAz))*CosElev(ICosElev)*xt...
                    +sin(AzimuthRadians(IAz))*CosElev(ICosElev)*yt)*Frequencies))).^2/N;
                %Gain is narrowband array gain as a matrix arranged as
                %azimuth (row #) x frequency (column #) x cosine(elevation angle)
            end
        end
%Broadband Array Gain Calculations
    %Gain will be computed as a function of bandwidth and center frequency.
        CenterFreq=Band(1)+ FIncr*(0:NCenterFreq-1);
    %Tested bandwidths will be odd multiples of FIncr.  For FIncr=0.1 MHz,
```

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%the "bandwidths" will correspond to sums of results for 1 frequency, 3
%frequencies, 5 frequencies, etc--having actual bandwidths (between
%outer frequencies) of 0, 0.2, 0.4, ... MHz.  However we ill consider,
%for example a sum across three measurements separated by 0.1 MHz to
%represent a bandwidth of 0.3 rather than 0.2 MHz.
    Bandwidth=FIncr*(1:2:1+2*Ifreqext)';
%Create an array to repeatedly compute broadband gain vs azimuth and center frequency for each bandwidth:
    BBGainAtAz=zeros(NAngleSteps,length(CenterFreq),length(CosElev));
%Create two arrays to store broadband gain vs bandwidth and center frequency:
%one maximized across azimuth at 0 elevation angle and one maximized
%across both azimuth and elevation angle (0-45 deg)
    BBGainAt0Elev=zeros(length(Bandwidth),length(CenterFreq));
    BBGain=zeros(length(Bandwidth),length(CenterFreq));
for Ibandwidth=1:Ifreqext
    ['Ibandwidth = ' int2str(Ibandwidth) ' of ' int2str(Ifreqext)]
    for ICenterFreq=1:length(CenterFreq);
        ICenter=ICenterFreq+Ifreqext;
      %Average the gain across bandwidth for each azimuth, center frequency, and elevation:
          %BBGainAtAz is organized as Azimuth X Center Freq X Cos(Elev)
          BBGainAtAz(:,ICenterFreq,:)=mean(NBGainAtAz(:,ICenter-Ibandwidth:ICenter+Ibandwidth,:),2);
    end
    %Find the maximum gain across azimuth at 0 deg elev for each frequency:
      BBGainAt0Elev(Ibandwidth,:)=max(BBGainAtAz(:,:,1),[],1);
    %Find the maximum gain across azimuth and elev for each frequency:
      BBGain(Ibandwidth,:)=max(max(BBGainAtAz,[],3),[],1);
end
%Select Excel spreadsheet filename for output:
    xlsFilename=['CDDArrayGainX' int2str(N)]
    Ibandwidth=83 %83 for FInc=0.1*****************
    xlswrite(xlsFilename,...
        [0,Bandwidth(1),Bandwidth(1),Bandwidth(Ibandwidth),Bandwidth(Ibandwidth);...
        CenterFreq',BBGain(1,:)',BBGainAt0Elev(1,:)',BBGain(Ibandwidth,:)',BBGainAt0Elev(Ibandwidth,:)'],...
        'GainVsFreq'); %Name of spreadsheet tab
%Find the maximum and median broadband gain across frequencies for each bandwidth and convert to dB:
    MaxBBGain=10*log10(max(BBGain,[],2));
    MedianBBGain=10*log10(median(BBGain,2));
    MaxBBGainAt0Elev=10*log10(max(BBGainAt0Elev,[],2));
    MedianBBGainAt0Elev=10*log10(median(BBGainAt0Elev,2));
%Write results to spreadsheet
    xlswrite(xlsFilename,...
        [Band',0,0,0; ...
        Bandwidth,MaxBBGain,MedianBBGain,MaxBBGainAt0Elev,MedianBBGainAt0Elev], ...
        'GainVsBW');
```