Auctionomics/Power Auctions Option for Forward Auction

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Introduction

This document supplements Appendix C to the September 28, 2012 NPRM, titled: "Incentive Auction Rules Option and Discussion." It provides additional details about several of the more novel aspects discussed in connection with the Forward Auction, including the use of intra-round bidding, managing the shifting of demand between categories of licenses and the forward auction closing rule.

We first offer a non-technical characterization of the main problems that the forward auction algorithm described below is designed to solve. There are three such issues.

A <u>first</u> issue in a standard clock auction is price overshooting. As an example, suppose that there are *five* licenses for sale in some category and that at the current price of 100, the total demand across all bidders is for *six* licenses. Suppose that at the next round the price is raised to 105 and the total demand at that price falls to just *four*: raising the price to 105 *overshoots* the market-clearing price. Because large increments make the substantial price overshooting more likely, standard clock auctions typically employ small bid increments that require more rounds and longer times to completion.

The design described in this document uses intra-round bidding to allow larger price increments in a round without the risk of price overshooting, thus resolving this first problem. With intra-round bidding, each bidder can specify the price point(s) at which it wishes to reduce its demand. In the example of the preceding paragraph, it might be that one bidder would specify that it wishes to reduce its demand by one at a price of 102 and another would specify the same at a price of 104. Our algorithm would determine that the clock should stop ticking up at a price of 102, when demand falls to be just equal to the supply. The reduction planned for the price of 104 would then not be implemented, so demand would be exactly equal to supply at the end of the round. Generally, if a round begins with aggregate demand at least as high as the target supply for a license category, our proposed intra-round bidding rules ensure that the round also ends with demand at least as high as supply. Intra-round bidding is illustrated by an example later in this document.

A <u>second</u> problem that can arise in clock auctions (regardless of whether they incorporate intra-round bidding) arises when a bidder proposes to increase demand for one product category while reducing demand for another. In a standard clock auction, the auctioneer cannot distinguish whether the bidder wishes to increase one demand *only if* it can reduce the other, or whether the bidder wishes to make

those changes independently. The proposed design remedies that problem by allowing packaging of demand changes. To illustrate, we revisit the example of the preceding paragraph but suppose that the second bidder, with price limit 104, is also interested in buying an additional license in second category, but does not have sufficient authorized budget to buy both. Suppose that the beginning and end of round prices for the second category are 80 and 84. In the proposed auction, a bidder's intra-round bid would specify that at the 80% price point (when the prices reach 104=100+.8x(105-100) for the first category and 83.2= 80+.8x(84-80) for the second category, it would reduce its demand for the first category and increase its demand for the second category. The proposed rules package changes at a single price point, so the increased demand for the second category would apply only if the bidder is permitted to reduce demand for the first category. In the example, the reduced demand for the first category is rejected, so that increased demand for the second category is not applied. In the same example, if the bidder wishes to delink the two changes, then it can make them at price points of 80% and 81%. Since the bids apply at different price points, the two would then be evaluated separately and the demand increase for the second category would be applied regardless of any restriction of the reduction for the first category.

A rule allowing bidders to specify combinations in this way is particularly valuable for this auction, in which there are multiple license categories and an activity rule. For example, suppose that a bid specifies reducing the quantities bid for categories A and B while increasing the quantities for categories C and D, but that the rules require that the reduction for A is rejected to avoid creating a situation in which demand falls below supply. Then, applying the activity rule, it may be that at least one of the increases must also be rejected, but which one? In the rule described here, it is the bidder itself who answers that question, as is necessary to promote efficiency of the final result. For example, if the bidder had bid to reduce the quantity for A and increase that for D at the 50% price point and to reduce the quantity for B and increase that for C at the 60% price point and the reduction for A were rejected, then the increase for D would also be rejected.

A third problem, which is novel in the context of the Incentive Auction, is that the supply of licenses in the reverse auction also depends on prices in the forward auction. In a standard clock auction, prices stop rising as soon as the demand falls to the target supply. In the Incentive Auction, such a rule could lead to prices that are too low to elicit the target supply from broadcasters, even if the bidders, if given another chance, would be willing to raise their bids enough to produce sufficient revenue to elicit the target supply. In the implementation described below, in such an event, prices continue to rise in the Forward Auction even when there is no excess demand relative to the target supply.

The algorithm described in the next section addresses each of the three problems described above. Choices about alternative rules, where applicable, have been made to encourage serious bidding and to allow the auction to progress as quickly as reasonably possible.

Overview of the Forward Auction

We assume here that the License Categories offered for sale will include "generic" licenses for paired spectrum (e.g. 5MHz uplink and 5MHz downlink) in each of the 176 Economic Areas (EAs). These EAs are the same ones that have been described and used in some previous FCC auctions. Generic licenses provide a right to use a specified amount of uplink and downlink bandwidth, but not to particular frequencies. The eventual band plan may also include other License Categories. For example, it may include downlink-only licenses, or it may include impaired licenses in some EAs. Impaired licenses may arise in EAs where less than the targeted amount of spectrum can be cleared at the reserve price. Downlink-only or impaired licenses in each EA would be separate License Categories in the Forward Auction. The rules we propose can also be adapted to other kinds of License Categories.

After the Forward Auction, there would be a frequency assignment process that assigns each winning bidder to particular (possibly contiguous) frequency blocks. The assignment process is not described in this document.

For the purposes of describing bids in the Forward Auction, "quantity" refers to a number of generic licenses in a License Category. If a bidder reduces its quantity demanded by one unit, that means it demands one fewer license.

The Closing Rule will determine the quantities in each License Category that are ultimately supplied during the final rounds of the Forward Auction. Because the broadcasters' willingness to sell their stations will depend on the prices offered, lower prices in the Forward Auction may result in a smaller quantity of licenses being ultimately supplied. Moreover, depending on decisions about the structure of the band plan, the quantities supplied may vary among EAs.

The proposed Forward Auction format is a multi-round clock auction. The auction relies on "intra-round" bidding that enables the bidding to proceed expeditiously in a discrete set of rounds, while giving the bidders flexibility to bid almost as if facing a series of very small price increases for the items in Excess Demand.

Under this approach, before each round, the FCC would announce an interval of prices that will be applicable to each License Category. Details of intra-round bidding are described below, but in overview, it proceeds as follows. During each round, each bidder may submit one or more bids. Each bid expresses changes in the quantities for each License Category that the bidder demands at a single "price point" in the announced interval. After each round, the FCC calculates the Aggregate Demand for licenses for each License Category and compares it to the Target Supply. If the Aggregate Demand exceeds the Target Supply, there is Excess Demand for that License Category. If there is Excess Demand for any License Category, then the prices for that License Category will be incremented in the next round; if there is Excess Demand for some License Categories but not for others, then the price for any License Category without Excess Demand is not incremented. If there is Excess Demand for any License Category, the auction proceeds to a next round and the process repeats.

Specific Forward Auction Rules

Except where explicitly stated otherwise, all references to "the auction" below are references to the Forward Auction.

Opening/Reserve Prices. The prices in "Round 0" of the auction are reserve prices set by the FCC. For each License Category r, the opening price is denoted by $p_{r,0}$.

Initial Bids. A bid in "Round 0" of the auction specifies a quantity for each License Category that the bidder is willing to purchase at the opening prices. A bidder may submit only one such bid in Round 0.

Price Intervals. For each round, t ($t \geq 1$), and for each License Category, r, the FCC announces to bidders an interval of prices, $[p_{r,t-1}, P_{r,t}]$. The low price of this interval is called the "start-of-round price" and the high price of this interval is called the "end-of-round price." The start-of-round price in Round 1, $p_{r,0}$, is the reserve price; the start-of-round price in Round t ($t \geq 2$) is the "posted price" of Round $t-1$ (which never exceeds the end-of-round price of Round $t-1$, so $p_{r,t} \leq P_{r,t}$).

Price Points. The "price point" is a parameter that indicates a price for every License Category. The price point indicates the percentage of the distance from the start-of-round price to the end-of-round price that is taken for each License Category. For example, the 0% price point refers to the start-of-round price, the 100% price point refers to the end-of-round price, and the 50% price point refers to the average of the start-of-round and end-of-round prices. The FCC could choose to allow intra-round bids to be submitted with any price points (e.g. from 0.01% to 100.00%), but no bid may be submitted at the price point 0.00%.

Intra-Round Bids. A bid in Round t ($t \geq 1$) of the auction comprises *changes* in quantity for one or more License Categories at a price point. For example, a bid could specify License Category A and a quantity of −1 at a price point X%; such a bid would specify a reduction of one unit at the indicated price point. Alternatively, a bid could specify License Category A and a quantity of −1, together with License Category B and a quantity of +1, at price point Y%; such a bid would specify a switch from License Category A to License Category B at the indicated price point. These bids, called "intra-round bids", are permitted to specify an arbitrary number of License Categories and quantities. A bidder may submit multiple intra-round bids in a given Round t ($t \geq 1$), which are applied cumulatively, but at most one bid at any price point. In each round, intra-round bids, when deemed feasible, are applied successively as changes to the bidder's *processed demands* (defined below) from the previous round.

Activity Rule. This rule requires bidders to meet an Activity Requirement during each round of the Forward Auction to maintain their bidding eligibility. Initial eligibility can be determined by bidder deposits before the start of the auction. In any round of the auction, a bidder's Eligibility is the smaller of (1) its Eligibility in the previous round or (2) the total number of points associated with its demand from the previous round divided by the Activity Requirement for that round.

Target Supply. For each round, t ($t \geq 1$), and for each License Category, r, the FCC announces to bidders a Target Supply, $S_{r,t}$.

Aggregate Demand. The Aggregate Demand for each License Category at any particular time during or after a round is the then-current sum of the demands of all bidders.

Feasibility of an Initial Bid. A bid in Round 0 is feasible if the total number of points associated with the bid does not exceed the bidder's Eligibility in Round 0.

Feasibility of an Intra-Round Bid. An intra-round bid in Round t is said to be *feasible* if application of all the changes specified in the bid results in demands that satisfy all of the following conditions:

  (i)   the total number of points associated with the bidder's demand (after the change) does not exceed the bidder's Eligibility in Round t; and

  (ii) for each License Category for which the change in demand specified by the intra-round bid is negative, the Aggregate Demand (including the bidder's demand after the change) is not less than the Target Supply.

Processing of Bids After a Round. After each round, intra-round bids are prioritized in the order of their price points: a bid at a lower price point receives a higher priority and ties among bids at the same price point are broken randomly. The highest-priority intra-round bid that has not yet been considered is the next bid to be processed. The *processing* of an intra-round bid includes checking its feasibility, where condition (ii) above is checked using the most-recently-determined demands of all bidders. If the intra-round bid is found to be feasible, then it is *applied*; otherwise, it is placed in a "queue" of all bids from Round t that have not been applied due to infeasibility. Whenever an intra-round bid is applied, the queue is then re-tested to determine whether any bids in the queue have become feasible; if so, the highest-priority feasible bid is applied and removed from the queue. The re-testing of the queue is iterated until no bids remaining in the queue are feasible. Then the next intra-round bid from Round t is processed, until all bids from the round have been processed and no bids in the queue are feasible. At the end of this processing, all bids remaining in the queue are discarded. The demands of the bidders following the processing of the intra-round bids for the round are referred to as the "*processed demands*."

Posting of Prices After Round t. For any License Category such that Aggregate Demand after processing exceeds the Target Supply, under this approach, the FCC would set the "posted price" equal to the end-of-round price for the round ($p_{r,t} = P_{r,t}$). For any License Category such that Aggregate Demand after processing does not exceed the Target Supply in that round and provided that some intra-round bid was applied that includes a decrease for that License Category, the FCC will set the "posted price" ($p_{r,t}$) equal to the price for that License Category at the highest price point of any intra-round bid that was applied in Round t and included a decrease for the License Category. For any License Category such that Aggregate Demand after processing does not exceed the Target Supply in that round and provided that no intra-round bid was applied that includes a decrease for that License Category, the

FCC will set the "posted price" ($p_{r,t}$) equal to the previous posted price ($p_{r,t-1}$) for that License Category.

Incrementing of Prices. For each License Category, the start-of-round price for Round *t* will be set equal to the Posted Price from the previous round ($P_{r,t} = p_{r,t-1}$) and the end-of-round price will be set at some amount (for example, between 1% and 20%) above the start-of-round price, depending on the extent of Excess Demand and on the round in the auction.

Supply Changes and Extended Rounds. At the end of a Round in which there is no Excess Demand – when the Aggregate Demand for *every* License Category is less than or equal to Target Supply for that License Category – the FCC tests to see whether the auction Closing Conditions have been met. If the Closing Conditions are met, the incentive auction ends.

If the Closing Conditions are not satisfied, then the FCC identifies those licenses in the Forward Auction for which Supplies would be reduced if the Clearing Target for channels in the reverse were reduced by some number. The Target Supplies of those licenses in the Forward Auction are reduced, but the Clearing Target remains unchanged.

The Forward Auction is continued in a series of Extended Rounds, in which prices are incremented based on the (now-reduced) Target Supplies. If after the processing of an intra-round bid in an Extended Round of the Forward Auction, the Closing Conditions are satisfied, then the overall auction ends and the current demands in the Forward Auction are filled at the then-current prices. At that time, demand for some licenses may exceed the reduced Target Supply in the Forward Auction but not the actual supply available from Reverse Auction.

If an Extended Round ends with no Excess Demand – meaning that the Aggregate Demand for *every* License Category is less than or equal to Target Supply for that License Category – the Extended Rounds end. The Clearing Target is then reduced (and action shifts to the reverse auction).

Bid Withdrawals. No bid in this auction may be withdrawn.

Information Policy. Before each round, the start-of-round price, the end-of-round price, the Target Supply for each License Category and the Activity Requirement will be announced to each bidder. After each round, the Posted Price, the Aggregate Demand (evaluated using the processed demands for that round) and the bidder's own processed demand for each License Category will be announced to each bidder.

Residue. It may happen, after the auction, that some licenses are unsold. This could happen, for example, if certain License Categories never received sufficient demand. Options for dealing with any residue are not described in this document.

Assignment Stage. After completion of the Forward Auction, which determines the number of units of each License Category allocated to each bidder, the Assignment Stage determines the actual frequencies to be assigned to each bidder in each EA based on the generic licenses won in the auction.

Example of the Intra-Round Bidding Algorithm

To illustrate the use of intra-round bids, we provide the following example with six License Categories.

License Category	A	B	C	D	E	F
Target Supply	9	9	9	9	9	9
Start-of-round price	40	20	40	60	40	20
End-of-round price	42	21	42	63	42	21

Suppose that the processed demands from the previous round are as follows:

Processed demands (end of previous round)	A	B	C	D	E	F
Bidder 1	1	1	1	0	1	1
Bidder 2	1	1	1	2	0	0
All others	8	9	8	10	12	10
Aggregate Demand	10	11	10	12	13	11

And suppose that the following intra-round bids are submitted in the current round:

Bids	Price point	A	B	C	D	E	F
Bidder 1	20%	−1	0	0	0	0	0
Bidder 2	40%	0	−1	0	0	0	0
Bidder 2	60%	−1	0	+1	0	0	0
Bidder 2	80%	0	0	0	−1	+1	+1

The first intra-round bid to be processed is Bidder 1's bid at the 20% price point, which is understood to be a proposed change in its demand from the 0% price point.

Demands	Price point	A	B	C	D	E	F
Bidder 1	0%	1	1	1	0	1	1
	20%	−1	0	0	0	0	0
Bidder 1	20%	0	1	1	0	1	1
Bidder 2	20%	1	1	1	2	0	0
All others	20%	8	9	8	10	12	10
Aggregate	20%	9	11	10	12	13	11

Observe that Bidder 1's intra-round bid is feasible, as its application would result in Aggregate Demand that is at least the Target Supply of 9 for all License Categories.

Next, Bidder 2's first bid, at the 40% price point, is processed as follows, noting that its processed demand from the previous round is carried forward and treated as its demand at the 0% price point of the current round. Meanwhile, Bidder 1's demand is unchanged from the 20% price point, and all other bidders' demands are unchanged from the previous round:

Demands	Price point	A	B	C	D	E	F
Bidder 2	0%	1	1	1	2	0	0
	40%	0	−1	0	0	0	0
Bidder 2	40%	1	0	1	2	0	0
Bidder 1	40%	0	1	1	0	1	1
All others	40%	8	9	8	10	12	10
Aggregate	40%	9	10	10	12	13	11

Observe that Bidder 2's bid at the 40% price point is feasible, as it results in Aggregate Demand that is at least the Target Supply of 9 in all License Categories.

Next, Bidder 2's bid at the 60% price point is processed as follows, noting that Bidder 1's demand is still unchanged from the 20% price point, and all other bidders' demands are still unchanged from the previous round:

Demands	Price point	A	B	C	D	E	F
Bidder 2	40%	1	0	1	2	0	0
	60%	−1	0	+1	0	0	0
Bidder 2	60%	0	0	2	2	0	0
Bidder 1	60%	0	1	1	0	1	1
All others	60%	8	9	8	10	12	10
Aggregate	60%	8	10	11	12	13	11

Observe that Bidder 2's bid at the 60% price point is *infeasible*, as it would result in Aggregate Demand that is only 8—less than the Target Supply of 9—for License Category A. Consequently, no part of this bid is implemented, neither the decrease for License Category A nor the increase for License Category C. Instead, Bidder 2's bid at the 60% price point is placed in the queue.

Finally, Bidder 2's bid at the 80% price point is processed. The first processing step is adding its bid to its demand at the 40% price point, having rejected its bid at the 60% price point. Thus, the calculations are:

Demands	Price point	A	B	C	D	E	F
Bidder 2	40%	1	0	1	2	0	0
	80%	0	0	0	−1	+1	+1
Bidder 2	80%	1	0	1	1	1	1
Bidder 1	80%	0	1	1	0	1	1
All others	80%	8	9	8	10	12	10
Aggregate	80%	9	10	10	11	14	12

After this (or any) intra-round bid is processed, another pass is made through bids in the queue to see if any can be feasibly applied. In this case, the one bid in the queue remains infeasible. There are no more bids to process, so bids in the queue are discarded and the round processing is over.

At the end of the round, for License Category A, Target Supply equals Demand. Since the last price point at which demand changed for this License Category was 20%, the Posted Price at the end of the round is 40.4, computed as 40+.2x(42-40). For all other License Categories, Demand exceeds Target Supply, and so the Posted Prices for those License Categories are equal to the end-of-round prices. The start-of-round prices for the next round are therefore given as follows:

License Category	A	B	C	D	E	F
Target Supply	9	9	9	9	9	9
Next start-of-round price	40.4	21	42	63	42	21

In summary, in this example, Bidder 1's bid, and two out of three of Bidder 2's bids, are applied; the remaining bid of Bidder 2 is not applied (and is ultimately discarded); and the price for License Category A rises by only 20% of the maximum increment for License Category A.