Supplemental Report of Professor Simon J. Wilkie

Comcast-NBCU Merger (MB Dkt. 10-56)

September 30, 2010

I. Introduction

1. This report supplements Professor Wilkie's June 21, 2010 Report, "Consumer Sovereignty, Disintermediation, and the Economic Impact of the Proposed Comcast/NBCU Transaction" ("Wilkie Report") and his August 19, 2010 Reply Report, "Economic Analysis of the Proposed Comcast-NBCU Merger" ("Wilkie Reply Report").

2. In the Wilkie Report, Professor Wilkie demonstrated that the merged Comcast/NBCU will have strong incentives to discriminate in favor of its own programming and "the post merger Comcast entity will have the incentive to raise the price of stand-alone broadband service absent other competitive pressures." Wilkie Report, at 8-13, 22-24. This merger-specific finding is significant because a rise in the price of Comcast stand-alone broadband service will harm consumer welfare by: (i) forcing some existing stand-alone broadband subscribers to drop their broadband service; (ii) increasing the charges to subscribers who wish to acquire online video distribution services via stand-alone broadband (and avoid Comcast's higher-priced bundled offerings) and "cut the cord"; and (iii) harming

the market for competitive Internet Service Providers by raising the price of access to end users.

3. In the Wilkie Reply Report, Professor Wilkie rebutted criticism from Professors Katz and Israel concerning the impact of the merger on the price of Comcast broadband stand-alone service. In oral testimony before the Commission staff on August 27, 2010, however, Professor Katz contended that Professor Simon's relevant analysis in the Wilkie Reply Report was mistaken. The attached Technical Addendum of the Wilkie Report ("Technical Addendum") demonstrates in greater detail how, in fact, the Comcast/NBCU merger will reduce consumer welfare by raising Comcast's incentives to increase the price of stand-alone broadband service.

II. Professor Wilkie's Economic Model

4. In the attached Technical Addendum, Professor Wilkie presents an economic model in which a single seller can sell two products, X (stand-alone broadband) and Y (cable television), either separately or together as a bundle. Under this model, the seller wishes to set three prices – for X, Y, and the bundle – in order to maximize profits. Consumers have four choices: (1) purchase X, (2) purchase Y, (3) purchase the bundle, and (4) do not purchase any of the products. The proportion of consumers who make any one of these four specific buying decisions is represented by a mathematical function that depends on the prices of the products.

5. The model can be solved for market equilibrium. The mathematical expressions completely characterize the set of prices at which the seller maximizes profit. Using these expressions, we may perform comparative statics by examining how a change in a parameter of the model affects an equilibrium result. For example, we can answer the question, "How does a decrease in the marginal cost of producing Y affect the equilibrium price of X?" Professor Wilkie's results demonstrate that when the marginal cost of producing cable services decreases, the equilibrium stand-alone price of broadband service increases. With access to the NBCU content and assets and increased efficiencies, the merger will decrease Comcast's overall costs of providing cable services. Therefore, as demonstrated by Professor Wilkie's economic model, the merger will increase Comcast's incentives to raise prices of stand-alone broadband services in contravention of consumer welfare and the public interest.

III. STRUCTURAL SOLUTION

6. Professor Wilkie has determined that the structural solution proposed by EarthLink – wholesale stand-alone broadband access – will mitigate this harm to the public interest and ensure that consumers benefit from the Comcast/NBCU transaction. As explained in the Wilkie Report, "[s]ufficient competitive choices, such as independent ISP like EarthLink could provide, would mitigate the harm to consumers who wished to remain with a stand-alone broadband provider at the old prices. This is because the availability of sufficient neutral provider choices would serve to discipline Comcast's ability to raise prices." Wilkie Report at 23-

24. Moreover, in the Wilkie Reply Report, Professor Wilkie concluded that "[a]llowing consumers to have a choice of ISPs will (1) let consumers 'break the bundle,' (2) promote competition and discipline Comcast's pricing, and (3) protect the development of the nascent online video market." Wilkie Reply Report at 29.

7. As described in the Wilkie Report, a condition requiring Comcast to sell wholesale stand-alone broadband access to independent ISPs has the added benefits of "indirectly encouraging the further development of online video programming by 'leveling the playing field.'" By increasing the difficulty for content-integrated ISPs to discriminate against non-affiliated programming, Professor Wilkie explains that "this solution will promote the growth and health of these programmers, giving consumers more diversity in online content. This will put pressure on Comcast to continue to invest in, and expand, their broadband network." Further, Professor Wilkie asserts the condition will "diminish the incentive of Comcast/NBCU to paralyze online video programming with unaffiliated content, as there would be a real marketplace 'penalty' imposed upon them by upset customers who will switch to another ISP." Wilkie Report at 47. As such, the Commission should consider this solution to remedy the merger-specific harms identified in the Technical Addendum.

TECHNICAL ADDENDUM TO THE WILKIE REPORT

In this Technical Addendum we explain in detail the claim made in a previous filing by Professor Simon Wilkie (filed June 21, 2010) that the merger of Comcast and NBCU is likely to raise the price of stand-alone broadband internet service.

The key assumptions are (i) that the profit margin of the bundled cable internet product exceeds the profit margin on the stand-alone products and (ii) that the densities of the values of both goods are log concave. In particular, given a density function f and its cumulative distribution function F, we use the assumptions that both $F(x)/f(x)$ is monotone increasing and $(1 - F(x))/f(x)$ is monotone decreasing. We know from the Israel and Katz reports that the first assumption is satisfied in the case at hand. As to the second assumption, these properties are held by the most commonly used distributions in Economics to model discrete demand, including the uniform, normal, and exponential distributions. For a detailed presentation of the properties of log-concave distributions, see Karlin and Rinott (1980), Bagnoli and Bergstom (2005), and Miravete (2010).[1] Bagnoli and Bergstrom provide a detailed list of distributions that satisfy these requirements.

This addendum presents the theoretical analysis and provides further numerical analysis of bundling for the cases of the uniform and exponential distributions. The uniform case is of interest because it is used widely as a benchmark and admits a closed-form solution. The exponential case is frequently used in empirical literature, and is of particular interest because it is both log concave and log convex. It is therefore the boundary case as the "least log concave" density function. In this addendum, we follow the terminology and approach of McAfee, McMillan, and Whinston (1989) and refer to *pure bundling* when the firm only sells the bundle and *mixed bundling* when the firm sells both the bundle and the individual goods; when we consider the case of mixed bundling, we refer to the prices of the individual goods as *stand-alone prices*.[2]

[1]Samuel Karlin and Yosef Rinott (1980), "Classes of Orderings of Measures and Related Correlation Inequalities, I. Multivariate Totally Positive Distributions," *Journal of Multivariate Analysis*, Vol. 10, pp. 467-498. Mark Bagnoli and Ted Bergstrom (2005), "Log Concave Probability and its Applications," *Economic Theory*, Vol 26, pp. 445-469. Eugenio Miravete (2010), "Aggregation of Totally Positive Distributions in Economics," Working Paper, University of Texas, Austin.

[2]R. Preston McAfee, John McMillan, and Michael D. Whinston (1989), "Multi-Product Monopoly, Commodity Bundling, and Correlation of Values," *Quarterly Journal of Economics*, Vol. 114, pp. 371-384.

A firm sells products A, B, and the bundle of A and B, whose prices are p_A, p_B, and p_{AB}. To avoid trivial cases, assume that, $p_A + p_B > p_{AB}$, $p_{AB} > p_A$, and $p_{AB} > p_B$. The seller chooses prices to maximize profit.

$$\max_{p_A, p_B, p_{AB}} \Pi = r_A (p_A - c_A) + r_B (p_B - c_B) + r_{AB} (p_{AB} - c_A - c_B)$$

where r_A, r_B, and r_{AB} are the demands for A, B, and the bundled good. c_A and c_B are the marginal costs of A and B.

Assume that a consumer's valuation of A and B, (v_A, v_B), follows a distribution with density $h(v_A, v_B)$. The consumer's utility is measured by the difference of the valuation he obtains and the price he pays. For simplicity, assume that for each good, a consumer can buy at most one unit. Therefore, the following cases are possible.

Case 1 A consumer only buys one unit of A.

Case 2 A consumer only buys one unit of B.

Case 3 A consumer buys one unit of A and one unit of B.

Case 4 A consumer buys nothing.

Assume that $h(v_A, v_B) = f(v_A) g(v_B)$ for $v_A \in [\underline{a}, \bar{a}]$ and $v_B \in [\underline{b}, \bar{b}]$, where $f(\cdot)$ and $g(\cdot)$ are marginal densities of v_A and v_B, respectively. Assume $F(\cdot)$ and $G(\cdot)$ are the corresponding distribution functions.

In Case 1,

$$\begin{aligned} v_A - p_A &> v_B - p_B \\ v_A - p_A &> v_A + v_B - p_{AB} \\ v_A - p_A &> 0 \end{aligned}$$

which jointly imply that

$$v_A > p_A \text{ and } v_B < p_{AB} - p_A.$$

Therefore,

$$\begin{aligned} r_A &= \int_{\underline{b}}^{p_{AB}-p_A} g(v_B) \int_{p_A}^{\bar{a}} f(v_A)\, dv_A dv_B \\ &= G(p_{AB}-p_A) - F(p_A)\, G(p_{AB}-p_A) \end{aligned}$$

In Case 2,

$$\begin{aligned} v_B - p_B &> v_A - p_A \\ v_B - p_B &> v_A + v_B - p_{AB} \\ v_B - p_B &> 0 \end{aligned}$$

which imply

$$v_A < p_{AB} - p_B \text{ and } v_B > p_B.$$

Therefore,

$$\begin{aligned} r_B &= \int_{p_B}^{\bar{b}} g(v_B) \int_{\underline{a}}^{p_{AB}-p_B} f(v_A)\, dv_A dv_B \\ &= F(p_{AB}-p_B) - F(p_{AB}-p_B)\, G(p_B) \end{aligned}$$

In Case 3,

$$\begin{aligned} v_A + v_B - p_{AB} &> v_A - p_A \\ v_A + v_B - p_{AB} &> v_B - p_B \\ v_A + v_B - p_{AB} &> 0 \end{aligned}$$

which imply

$$\begin{aligned} v_A &> p_A \text{ and } v_B > p_{AB} - p_A, \text{ OR} \\ p_{AB} - p_B &< v_A < p_A \text{ and } v_B > p_B, \text{ OR} \\ v_A &< p_A, v_B < p_B \text{ and } v_A + v_B > p_{AB}. \end{aligned}$$

Therefore,

$$\begin{aligned}
r_{AB} &= \int_{p_{AB}-p_A}^{\bar{b}} g\left(v_B\right) \int_{p_A}^{\bar{a}} f\left(v_A\right) dv_A dv_B + \int_{p_B}^{\bar{b}} g\left(v_B\right) \int_{p_{AB}-p_B}^{p_A} f\left(v_A\right) dv_A v_B + \\
&\int_{p_{AB}-p_A}^{p_B} g\left(v_B\right) \int_{p_{AB}-v_B}^{p_A} f\left(v_A\right) dv_A dv_B \\
&= 1 - G\left(p_{AB}-p_A\right) - F\left(p_{AB}-p_B\right) + F\left(p_{AB}-p_B\right) G\left(p_B\right) \\
&\quad - \int_{p_{AB}-p_A}^{p_B} g\left(v_B\right) F\left(p_{AB}-v_B\right) dv_B
\end{aligned}$$

To solve the profit-maximization problem $\Pi(p_A, p_B, p_{AB})$, the first order conditions (FOCs) are presented below. To shorten notation, we will denote p_A, p_B, p_{AB}, $p_{AB}-p_A$, and $p_{AB}-p_B$ by x, y, z, u, and w respectively.

$$\begin{aligned}
p_A &: \frac{\partial r_A}{\partial p_A}\left(p_A - c_A\right) + r_A + \frac{\partial r_{AB}}{\partial p_A}\left(p_{AB} - c_A - c_B\right) = 0 \Longrightarrow \qquad (1) \\
& G\left(u\right)\left[1 - F\left(x\right)\right] - f\left(x\right) G\left(u\right)\left(x - c_A\right) - \\
& \left[1 - F\left(x\right)\right] g\left(u\right)\left(x - c_A\right) + g\left(u\right)\left[1 - F\left(x\right)\right]\left(z - c_A - c_B\right) \\
&= 0
\end{aligned}$$

$$\begin{aligned}
p_B &: \frac{\partial r_B}{\partial p_B}\left(p_B - c_B\right) + r_B + \frac{\partial r_{AB}}{\partial p_B}\left(p_{AB} - c_A - c_B\right) = 0 \Longrightarrow \qquad (2) \\
& F\left(w\right)\left[1 - G\left(y\right)\right] - g\left(y\right) F\left(w\right)\left(y - c_B\right) - \\
& \left[1 - G\left(y\right)\right] f\left(w\right)\left(y - c_B\right) + f\left(w\right)\left[1 - G\left(y\right)\right]\left(z - c_A - c_B\right) \\
&= 0
\end{aligned}$$

$$\begin{aligned}
p_{AB} &: \frac{\partial r_A}{\partial p_{AB}}(p_A - c_A) + \frac{\partial r_B}{\partial p_{AB}}(p_B - c_B) + \frac{\partial r_{AB}}{\partial p_{AB}}(p_{AB} - c_A - c_B) + r_{AB} \qquad (3)\\
&= 0 \Longrightarrow \\
& [1 - F(x)]\, g(u)(x - c_A) + [1 - G(y)]\, f(w)(y - c_B) - \\
& \left\{ [1 - F(x)]\, g(u) + [1 - G(y)]\, f(w) + \int_u^y g(v_B)\, f(z - v_B)\, dv_B \right\} (z - c_A - c_B) + \\
& 1 - G(u) - F(w) + F(w)\, G(y) - \int_u^y g(v_B)\, F(z - v_B)\, dv_B \\
&= 0
\end{aligned}$$

As a simpler case, if $f(.) = g(.) \equiv h(.)$, we would have

$$\begin{aligned}
r_A &= H(u) - H(x)\, H(u) \\
r_B &= H(w) - H(y)\, H(w) \\
r_{AB} &= 1 - H(u) - H(w) + H(w)\, H(y) - \int_u^y h(v_B)\, H(z - v_B)\, dv_B
\end{aligned}$$

and the FOCs are

$$\begin{aligned}
p_A &: H(u)[1 - H(x)] - h(x)\, H(u)(x - c_A) - \\
& [1 - H(x)]\, h(u)(x - c_A) + h(u)[1 - H(x)](z - c_A - c_B) \\
&= 0
\end{aligned}$$

$$\begin{aligned}
p_B &: H(w)[1 - H(y)] - h(y)\, H(w)(y - c_B) - \\
& [1 - H(y)]\, h(w)(y - c_B) + h(w)[1 - H(y)](z - c_A - c_B) \\
&= 0
\end{aligned}$$

$$\begin{aligned}
p_{AB} &: [1 - H(x)]\, h(u)(x - c_A) + [1 - H(y)]\, h(w)(y - c_B) - \\
& \left\{ [1 - H(x)]\, h(u) + [1 - H(y)]\, h(w) + \int_u^y h(v_B)\, h(z - v_B)\, dv_B \right\} (z - c_A - c_B) + \\
& 1 - H(u) - H(w) + H(w)\, H(y) - \int_u^y h(v_B)\, H(z - v_B)\, dv_B \\
&= 0
\end{aligned}$$

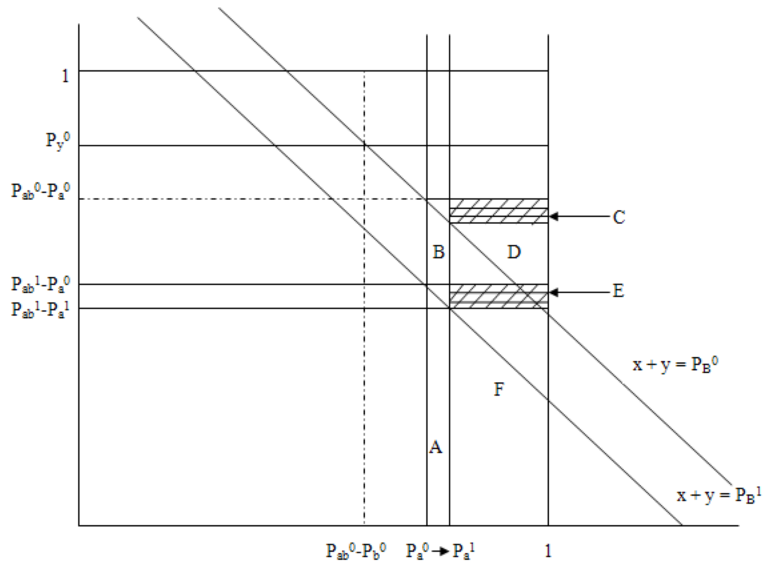

Figure 1: First Order Conditions

To gain insight into the first order conditions, consider Figure 1. We will use the letters A, B, C, D, E, and F as shorthand for the probability masses in the rectangles indicated. Thus for example, "A"$= [F(p_A^1) - F(p_A^0)]G(p_{AB}^1 - p_A^0)$, and "F" $= [1 - F(p_A^1)]G(p_{AB}^1 - p_A^1)$. Suppose that at the initial marginal costs, (c_A, c_B^0), initial optimal prices were (p_A^0, p_B^0, p_{AB}^0). We assume that when the marginal cost of B falls to $c_B^1 < c_B^0$, the new level of optimal prices are (p_A^1, p_B^1, p_{AB}^1) with $p_{AB}^1 < p_{AB}^0$; that is, we assume that merger efficiencies lead to a fall in the price of the bundle.

Now consider a small, i.e., epsilon (ϵ), increase in the price of A. By definition, at the initial optimum, the change in profits will be zero and thus marginal lost profits equal marginal gains. In Figure 1, this implies that:

$$(p_A^0 - c_A)(A + B) = \epsilon(C + D + E + F) + [(p_{AB}^0 - c_B^0 - p_A^0) - \epsilon]C,$$

and therefore that $(p_A^0 - c_A)(A + B) > \epsilon(C + D + E + F)$. Notice also, by construction, that the ratio A/(E+F) equals B/(C+D).

Therefore, if the mass of E is greater than that of C, it follows that $(p_A^0 - c_A)(A) > \epsilon(E + F) + [(p_B^0 - c_B^0 - p_A^0) - \epsilon]E$ when the incremental profit of the bundle is positive, i.e., when $(p_B^0 - c_B^0 - p_A^0) > 0$.

Note also that if the "pass through rate" (i.e., the change in monopoly price for a change in marginal cost) for the price of the bundle is less than 1, then $(p^0_{AB} - c^0_B) < (p^1_{AB} - c^1_B)$, and then:

$$(p^0_A - c_A)(A) > \epsilon(E + F) + [(p^0_B - c^0_B - p^0_A) - \epsilon]E.$$

Therefore, the partial derivative of profits with respect to p_A evaluated at (p^0_A, p^1_B, p^1_{AB}) is positive, and so the firm has the incentive to raise the stand-alone price of A. The required properties on demand to obtain the above result are related to the shape of the density functions f and g.

We return now to the limiting case and the first order conditions.

The FOC for p_A implies that:

$$\left[\frac{1 - F(p^0_A)}{f(p^0_A)} - (p^0_A - c_A)\right]\frac{G(u)}{g(u)} + \left[p^0_{AB} - c^0_B - p^0_A\right]\frac{1 - F(p^0_A)}{f(p^0_A)} = 0$$

Thus the FOC can written terms of mark up factors and the hazard functions. In particular, the log-concavity of densities f and g implies that (i) both $\frac{1-F(p_A)}{f(p_A)}$ is monotone decreasing and $\frac{G(u)}{g(u)}$ is monotone increasing - or in the limit, the size of E is greater than size of C; (ii) the distribution of the value of the bundle A+B (the convolution) is also log concave and this property is inherited on sub-domains; and (iii) that the pass through rate of the monopoly price is less than or equal to one.[3]

Notice that at the "stand-alone" monopoly price for A we have $(1 - F(p_A))/f(p_A) = (p_A - c_A)$. Thus, if the bundle has a higher profit margin than A, the optimal price under mixed bundling is greater than the stand-alone monopoly price. Thus the first square bracket term is negative and similarly the second square bracket term is positive. Now, consider a decrease in c_B, leading to a reduction in p_B. As before, if the pass

[3]See Mark Bagnoli and Ted Bergstrom (2005), "Log Concave Probability and its Applications," *Economic Theory*, Vol. 26, pp. 445-469, for the monotonicity results and the preservation of log concavity to sub-domains. See Eugenio Miravete (2010), "Aggregation of Totally Positive Distributions in Economics," Working Paper, University of Texas, Austin, for the properties of convolutions. See E. Glen Weyl and Michal Fabinger (2009), "Pass Through as an Economic Tool," Working Paper, Harvard University, for a discussion of log concavity and pass through rates.

through rate is less than one, then $(p^0_{AB} - c^0_B) < (p^1_{AB} - c^1_B)$. In addition, we have that $(p^0_{AB} - p^0_A) > (p^1_{AB} - p^0_A)$ and thus $G(u))/g(u)$ has fallen. Therefore, we can see that the reduction in p^0_{AB} implies that $\frac{\partial \Pi}{\partial p_A}(p^0_A, p^1_B, p^1_{AB}) > 0$.

Now, define $V_a^{\max}$ to be the maximum value that a consumer could have for good A when the distribution is bounded. Let $p^*_A = \min[p^1_{AB}, V_a^{\max}]$; as $(1 - F(p^0_A))/f(p^0_A)$ is positive and monotone decreasing in p_A, and $G(u))/g(u)$ is positive and monotone increasing in p_A, and $G(0) = 1 - F(V_a^{\max}) = 0$, it follows that $\frac{\partial \Pi}{\partial p_A}(p^*_A, p^1_B, p^1_{AB}) < 0$. Therefore, the solution to the first order condition is attained at some $p^1_A > p^0_A$. Thus if the cost reduction of good B- or an equivalent increase in the value to the firm of a "bundled" consumer leads to a lower price for the bundle, then the optimal price of the stand-alone good A will rise.

To illustrate the above claims, I present results for the symmetric, uniform, and exponential cases. Again, the exponential case is interesting as the boundary case - it is log linear and so is the "least log concave" case. In the exponential case, the pass through rate of a change in marginal cost on price is exactly one for both the stand-alone goods and the pure bundling case, but the pass-through rate for the price of the bundle under mixed bundling is less than one.

For the uniform [0,1] case, the first order conditions constitute a system of three quadratic equations. In the Excel simulations reported below, profit-maximizing prices were calculated on a pricing grid for various levels of the marginal costs. Since $0 \leq p_{AB} \leq p_A + p_B \leq 2$, we search the interval $[0, 2]$ for the optimal prices with grid-length of 0.0001. The optimal prices for each good and the bundle, as the marginal cost of B changes, are graphed below. The graphs confirm that the cross pass-through rate is negative: a lower marginal cost of A results in a higher price of B. The symmetric uniform case has recently been solved in Eckalbar (2010) (with symmetric costs) and a closed form analytic solution for the prices is now available.[4] Simulations are presented in Figure 2.

For the exponential case, first order conditions yield expressions in terms of hazard rates and product markups; we solve the first order conditions simulataneously using a mathematical software package (MATLAB) and plot results (see Figure 3).

[4]John Eckalbar (2010), "Closed Form Solutions to the Bundling Problem," *Journal of Economics and Management Strategy*, Vol. 19, pp. 513-544.

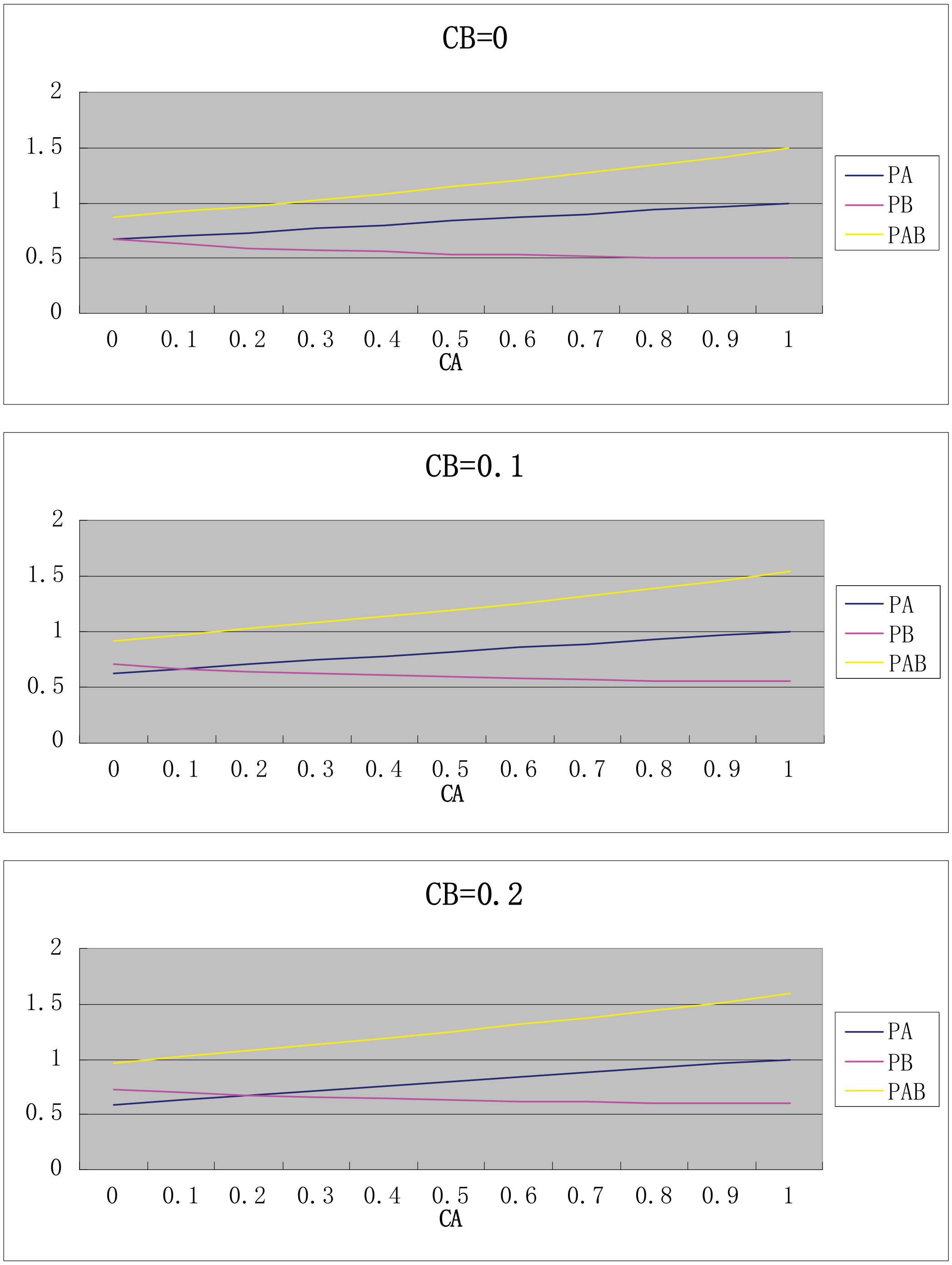

Figure 2: Simulations for the Symmetric Uniform Case

The Symmetric Exponential Case

Consider the case in which $F(\cdot) = G(\cdot) \equiv H(\cdot)$, where $H(\cdot)$ is the cumulative distribution of the exponential density function:

$$H(t) = \begin{cases} 1 - e^{-t}, & t \geq 0, \\ 0, & t < 0. \end{cases}$$

Denote the pdf of $H(\cdot)$ by $h(\cdot)$:

$$h(t) = \begin{cases} e^{-t}, & t \geq 0, \\ 0, & t < 0. \end{cases}$$

The solution space of prices is restricted to those values of $(x, y, z) \in \mathbb{R}^3$ satisfying $x, y, z \geq 0$, $x + y > z$, and $z > x, y$. Let $u = z - x$ and $w = z - y$. Then, in the region of interest to us, $h(j) = 1 - H(j)$ for $j \in \{x, y, z, u, w\}$. We also have $h(x)\, h(u) = e^{-x}e^{-(z-x)} = e^{-z}$ and $h(y)h(w) = e^{-y}e^{-(z-y)} = e^{-z}$. That is, the products $h(x)\, h(u)$ and $h(y)\, h(w)$ are both equal to e^{-z}.

Let $m_X = x - c_X$, $m_Y = y - c_Y$ and $m_Z = z - c_Z$: these are the markups of each good. Now, the first two FOCs yield:

$$e^{-x}(1 - m_X) = e^{-z}(1 - m_Z) \tag{4}$$

$$e^{-y}(1 - m_Y) = e^{-z}(1 - m_Z) \tag{5}$$

Now, rearrange the third FOC in the following way:

$$LHS = RHS,$$

where

$$\begin{aligned}
\text{LHS} =& 1 - H(u) - H(w) + H(w)H(y) - \int_u^y h(t)H(z-t)dt \text{ , and} \\
\text{RHS} =& -[1 - H(x)]h(u)m_X - [1 - H(y)]h(w)m_Y + \\
& \left\{[1 - H(x)]h(u) - [1 - H(y)]h(w) + \int_u^y h(t)h(z-t)dt\right\} m_Z
\end{aligned}$$

We work first with the LHS. Observe that

$$\begin{aligned}
\int_u^y h(t)H(z-t)dt &= \int_u^y h(t)(1 - h(z-t))dt \\
&= \int_u^y h(t)dt - \int_u^y h(t)h(z-t))dt \\
&= \int_u^y e^{-t}dt - \int_u^y e^{-z}dt \\
&= -[e^{-t}]_u^y - e^{-z}[1]_u^y \\
&= -(e^{-y} - e^{-u}) - e^{-z}(y-u) \\
&= -[e^{-y} - e^{-u} + e^{-z}(y-u)] \\
&= -[h(y) - h(u) + e^{-z}(y-u)]
\end{aligned}$$

Thus, we have

$$\begin{aligned}
\text{LHS} &= 1 - H(u) - H(w) + H(w)H(y) - \int_u^y h(t)H(z-t)dt \\
&= \{1 - H(u)\} - H(w) + H(w)H(y) + [h(y) - h(u) + e^{-z}(y-u)] \\
&= h(u) - H(w)(1 - H(y)) + h(y) - h(u) + e^{-z}(y-u) \\
&= -H(w)h(y) + h(y) - +e^{-z}(y-u) \\
&= h(y)[1 - H(w)] + e^{-z}(y-u) \\
&= h(y)h(w) + e^{-z}(y-u) \\
&= e^{-z}(1 + y - u) \qquad (6)
\end{aligned}$$

On the right hand side, we have:

$$\begin{aligned}
\text{RHS} &= -[1-H(x)]h(u)(x-c_Y) - [1-H(y)]h(w)(y-c_Y)+ \\
&\left\{[1-H(x)]h(u) + [1-H(y)]h(w) + \int_u^y h(t)h(z-t)dt\right\}(z-c_X-c_Y) \\
&= -e^{-z}m_X - -e^{-z}m_Y + 2e^{-z}m_Z + e^{-z}(y-u)m_Z \qquad (7)
\end{aligned}$$

Equating (6) and (7), we have

$$\begin{aligned}
e^{-z}(-m_X - m_Y + m_Z + m_Z(1+y-u)) &= e^{-z}(1+y-u) \\
\{m_Z - (m_X + m_Y)\} - (1-m_Z)(1+y-u)) &= 0 \qquad (8)
\end{aligned}$$

Equations (4), (5) and (8) characterize market equilibrium.

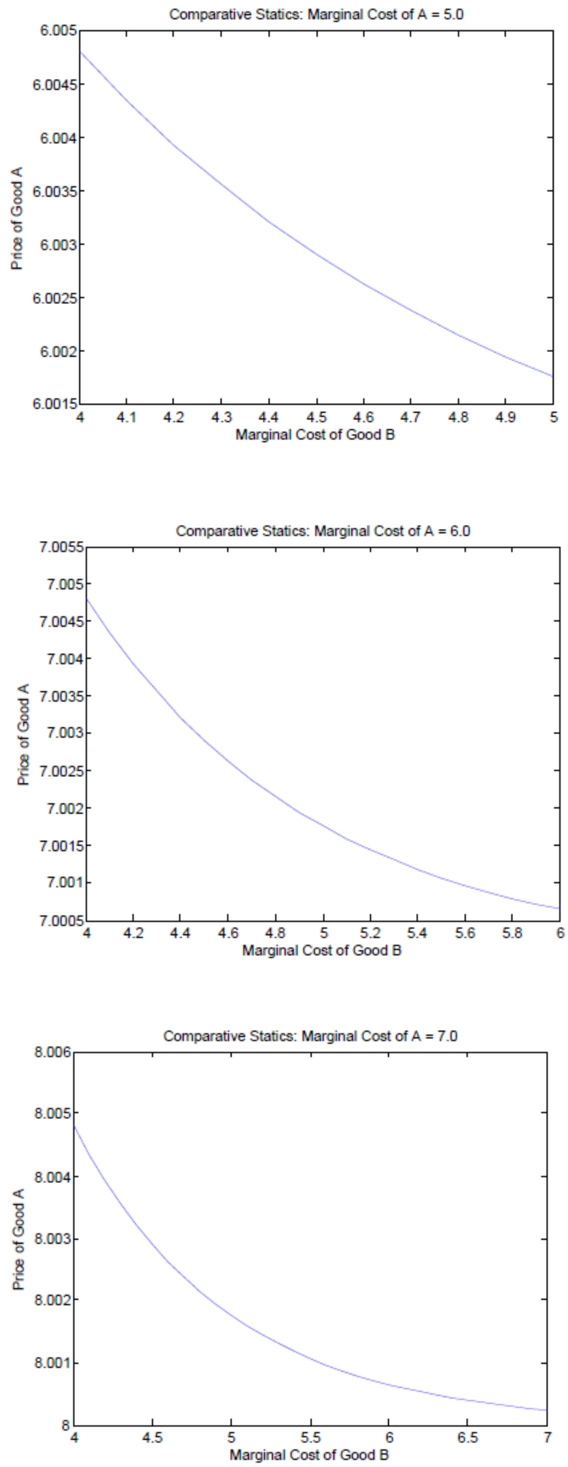

Figure 3: Simulations for the Symmetric Exponential Case